Reservoir Characterization from Generalized Well Test Data

With the Aid of System Identification Techniques

Mehdi Mansoori\textsuperscript{1,2}, P.M.J. Van den Hof\textsuperscript{1,4}, J. D. Jansen\textsuperscript{3}, D. Rashtchian\textsuperscript{2}

1: Delft Center for Systems and Control, Delft University of Technology, the Netherlands
2: Dept. of Chemical & Petroleum Eng., Sharif University of Technology, Iran
3: Dept. Geoscience & Engineering, Delft University of Technology, the Netherlands
4: Dept. of Electrical Engineering, Eindhoven University of Technology, the Netherlands
Oil and Gas Reservoir

- Reservoir ~ porous rock
- Depth ~ 1000-4000 m
- Long term production life ~ 20 - 100 year

- A **model** is essential to manage reservoir development and oil production
The Reservoir Model

- **Permeability** distribution determines flow pattern.
- Estimate the average permeability around the wells by **well test**.

\[ q(t) \propto k \frac{dp}{dx} \]
Conventional Well Test

- Manipulating the flow rate at surface.
- Response contains flow period responses.
- Pressure and flow variations are related by pressure transient analysis.
Steps in Conventional Well Test Analysis

- Log-Log plot for pressure and its derivative for (biggest) BU flow period
- Model selection (type curve matching)
- Parameter estimation

- Only one shut-in period is used.
- Loss of production.
- Effect of wellbore storage model in the data masks the reservoir response.
Data From a PDG

- PDG data is recorded under uncontrolled circumstances.
- Downhole pressure data for months (even years)
- Information of the reservoir (no wellbore dynamic)
The research problem in well test:

Given a set of measured (noisy) $p_{mbh}$, and variable (noisy) $q_{mbh}$

1) Identify the reservoir system
2) Estimate the physical parameter.

The classical system identification problem:

given (noisy) $u$, (noisy) $y$ of plant $G_0$ and possibly external signal $r$,

1) Identify a plant model $G$
   (for control, simulation or prediction)
Definition of the Problem

flow rate is varied

pressure is changed

$q_{wh} \rightarrow \text{wellbore} \rightarrow q_{bh} \rightarrow \text{Reservoir} \rightarrow p_{bh}$

$p$, $dp/d\ln t$

log(time)

 streamline

$BU_1 \rightarrow DD_1 \rightarrow DD_2$
Outline

permeability
wellbore damage

physical parameter estimation

reservoir

system identification

single well

causal structure

closed-loop

errors-in-variable

\[ G(q^{-1}) = \frac{B(q^{-1})q^{-(f+1)}}{F(q^{-1})} \]
Fluid Flow in the Wellbore

Mass conservation

Momentum conservation

B.C.:

Wellhead side: flow rate

Bottomhole side: pressure

Slightly compressible single phase fluid model

\[
\begin{bmatrix}
    Q_{bh}(s) \\
    P_{wh}(s)
\end{bmatrix} = W
\begin{bmatrix}
    Q_{wh}(s) \\
    P_{bh}(s)
\end{bmatrix}
\]
\[ Q_{bh}(s) = W_{11}(\beta, s)Q_{wh} + W_{12}(\beta, s)P_{bh}(s) \]

\[ P_{wh}(s) = W_{21}(\beta, s)Q_{wh} + W_{w2}(\beta, s)P_{bh}(s) \]
Fluid Flow in the Reservoir

Diffusivity equation

Darcy’s Law

B.C.:
- Sandface side: flow rate
- Outer edge side: pressure

\[
\begin{bmatrix}
Q_e(s) \\
P_{sf}(s)
\end{bmatrix}
= \mathbb{R}
\begin{bmatrix}
Q_{sf}(s) \\
P_e(s)
\end{bmatrix}
\]
\[ P_{sf}(s) = R_{21}(\beta, s)Q_{sf} + R_{22}(\beta, s)P_e(s) \]

\[ Q_e(s) = R_{11}(\beta, s)Q_{sf} + R_{12}(\beta, s)P_e(s) \]
Bilaterally Coupling the Systems
Well test measurements

- A Errors-In-Variable (EIV) in Closed Loop problem

- Not possible to identify R21 with $q^{mbh}$, $p^{mbh}$

Outline

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well damage  
porosity

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Outline

\[ G(q^{-1}) = \frac{B(q^{-1})q^{-f+1}}{F(q^{-1})} \]

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Outline

\[ G(q^{-1}) = \frac{B(q^{-1})q^{-f+1}}{F(q^{-1})} \]
Two-stage closed-loop identification

First open loop identification

\[ q_{bh}^m(k) = G_{fs}(q^{-1}, \theta_0)q_{wh}(k) + w_q(k) \quad k = 0, \ldots N \]

\[ \hat{q}_{bh0}(k) = G_{fs}(q^{-1}, \hat{\theta})q_{wh}(k) \quad k = 1, \ldots N. \]

Second open-loop identification

\[ p_{bh}^m(k) = R_{21}^d(q^{-1})\hat{q}_{bh0}(k) + w_p(k) \]
The Research Problem

permeability  
well damage  
porosity

physical parameter estimation

reservoir model

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errors-in-variable

closed-loop

\[
G(q^{-1}) = \frac{B(q^{-1})q^{-1}(f+1)}{F(q^{-1})}
\]
**Physical Parameters Estimation**

<table>
<thead>
<tr>
<th>Physic-based model</th>
<th>Data-based model</th>
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</thead>
<tbody>
<tr>
<td><strong>Time-domain</strong></td>
<td>discrete-time data</td>
</tr>
<tr>
<td>PDE equations</td>
<td></td>
</tr>
<tr>
<td><strong>Complex-domain</strong></td>
<td></td>
</tr>
<tr>
<td>$R_{21}(s) = R_{11}(s, k, S)$</td>
<td>$\hat{G}_{ss}(z, \hat{\theta}_N) = \frac{B(z, \hat{\theta}_N)}{F(z, \hat{\theta}_N)}$</td>
</tr>
<tr>
<td><strong>Frequency-domain</strong></td>
<td></td>
</tr>
<tr>
<td>$R_{21}(k, S, j\omega)$</td>
<td>$\hat{G}_{ss}(e^{j\omega}, \hat{\theta}_N)$</td>
</tr>
</tbody>
</table>

$$\hat{\beta} = \frac{1}{L} \arg\min_{\beta} \sum_{l=1}^{L} \left\| R_{21}(\beta, j\omega_l) - \hat{G}_{ss}(e^{j\omega_l}, \hat{\theta}_N) \right\| W(\omega_l)$$

$$R_{21} = \frac{\mu}{2\pi k hr_w} \sqrt{\frac{s}{\eta}} \frac{I_{0w}K_{0w} - I_{0w}K_{0e}}{I_{0e}K_{1w} + I_{1w}K_{0e}} + S$$

$\hat{G}_{ss}(z, \hat{\theta}_N) = \frac{B(z, \hat{\theta}_N)}{F(z, \hat{\theta}_N)}$
Case Studies

Case 1
Well: single-phase model

Case 2
Well: small WBS coef.

Case 3
Well: big WBS model
Case Studies

Case 4
Random input signal

Case 5
Twice noise level

Case 6
Positive skin factor
Identified Reservoir Models

\[ R_{21}(s) = \frac{\mu}{2\pi khr_w} \left[ I_0 \left( \frac{r_w}{\sqrt{\eta}} \right) K_0 \left( \frac{r_e}{\sqrt{\eta}} \right) - I_0 \left( \frac{r_w}{\sqrt{\eta}} \right) K_0 \left( \frac{r_e}{\sqrt{\eta}} \right) \right] + F \]

\[ \hat{G}_{ss}(z, \hat{\theta}_N) = \frac{B(z, \hat{\theta}_N)}{F(z, \hat{\theta}_N)} \]
Estimated reservoir parameters

Cases 1-5:
\[ K_{\text{true}} = 200 \text{ mD} \]
\[ \text{Skin}_{\text{true}} = 0 \]

Cases 6:
\[ K_{\text{true}} = 200 \text{ mD} \]
\[ \text{Skin}_{\text{true}} = 2 \]

<table>
<thead>
<tr>
<th>Table 5: Estimation Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability (mD)</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>case 1</td>
</tr>
<tr>
<td>case 2</td>
</tr>
<tr>
<td>case 3</td>
</tr>
<tr>
<td>case 4</td>
</tr>
<tr>
<td>case 5</td>
</tr>
<tr>
<td>case 6</td>
</tr>
</tbody>
</table>
Field data – An onshore gas well
Field data – TS SI method result
Field data – An onshore gas well
Field data – temperature measurement
Field data – EIVIV Method Results
### Physical Parameter Estimation Results

<table>
<thead>
<tr>
<th>Estimated physical parameters</th>
<th>Permeability (mD)</th>
<th>Skin</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS SI</td>
<td>10.6</td>
<td>-2.4</td>
</tr>
<tr>
<td>EIVIV</td>
<td>15.8</td>
<td>-1.4</td>
</tr>
<tr>
<td>Conventional PTA</td>
<td>11</td>
<td>-2.6</td>
</tr>
</tbody>
</table>
Conclusion

- Bilaterally coupled models increase the insight for the well test problem.
- System identification can effectively solve the well test analysis problem (no need to shut-in)
- The two-stage method can remove the wellbore effect.
- The physical estimation can be done in the frequency-domain in a well-defined way.
- Method can be generalized to treat noise on well-head flow measurements.
Thank you for your attention.