Tutorial Session: Data-driven modeling in dynamic networks

2021 European Control Conference – Session We TS

30 June 2021, 15:30 – 17:30 CEST

European Research Council

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Tutorial Session: Data-driven modeling in dynamic networks

Paul Van den Hof

2021 European Control Conference – Session We TS

30 June 2021, 15:30 – 17:30 CEST

www.sysdynet.eu
www.pvandenhof.nl

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Tutorial session: Data-driven modeling in dynamic networks

- Introduction (Paul Van den Hof) 15:30 – 16:00
  - Modeling framework, identification challenges, identifiability

- Graph-based method for analysing identifiability and allocating excitation signals (Xiaodong Cheng) 16:00 – 16:30

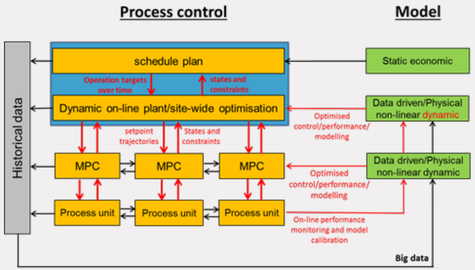
- Algorithm for full network identification and a case study in gas pipeline monitoring (Arne Dankers) 16:30 – 17:00

- Identification of single modules in a dynamic network (Karthik Ramaswamy) 17:00 – 17:30

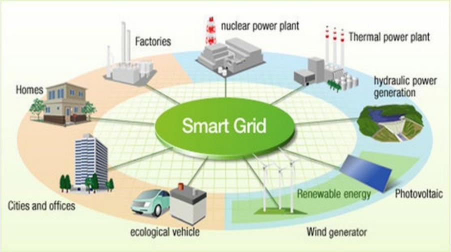
Feel free to raise questions in the Q&A

Introduction – dynamic networks

Decentralized process control

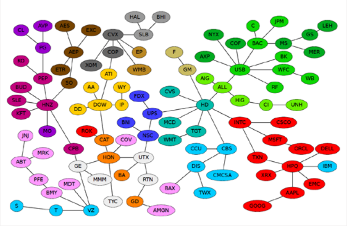


Smart power grid



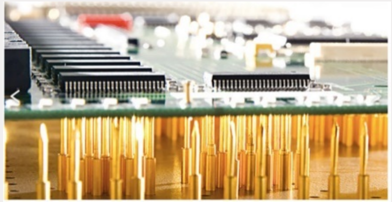
Betterworldsolutions.eu

Stock market



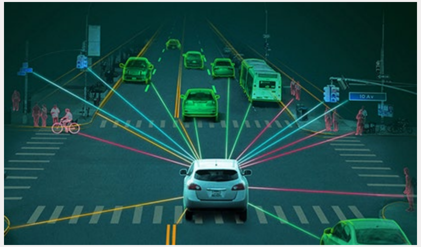
Materassi and Innocenti, 2010

PCB testing



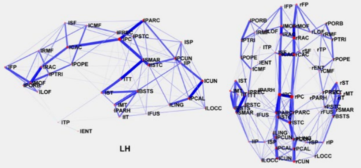
T&M Solutions, Romex BV

Autonomous driving



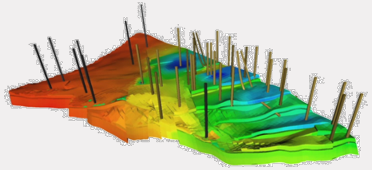
www.envidia.com

Brain network



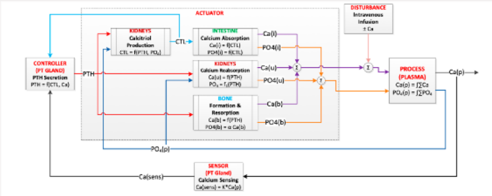
P. Hagmann et al. (2008)

Hydrocarbon reservoirs



Mansoori (2014)

Physiological models



Christie, Achenie and Ogunnaike (2014)

Introduction

Overall trend:

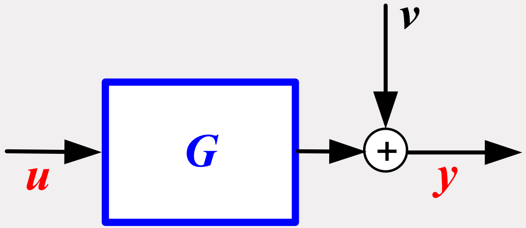
- Systems become more and more interconnected and large scale
- The scope of system's control and optimization becomes wider
  From components/units to systems-of-systems
- Modeling, monitoring, control and optimization actions become distributed
- Data is playing an increasing role in monitoring, decision making, control of (highly autonomous) smart systems (machine learning, AI)

- → **Learning models/actions from data** (including physical insights when available)

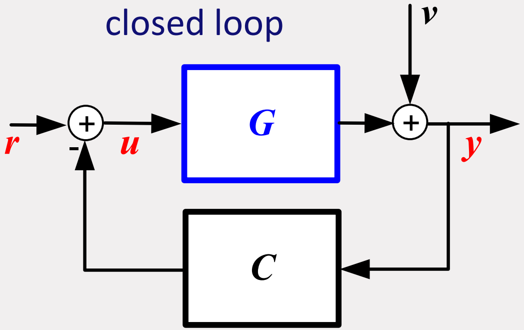
Introduction

The classical (multivariable) identification problems[1] :

open loop



closed loop

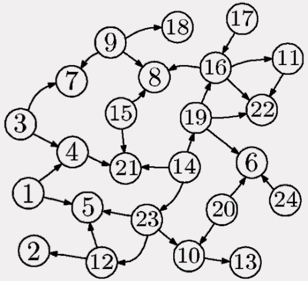


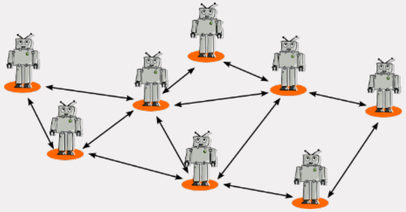
Identify a model of G on the basis of measured signals u, y (and possibly r), focusing on *continuous LTI dynamics*.

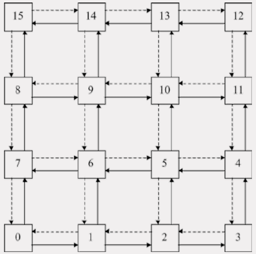
We have to move from a simple and fixed configuration to deal with ***structure*** in the problem.

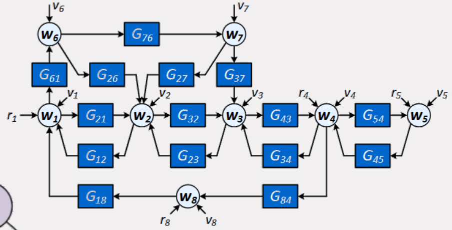
[1] Ljung (1999), Söderström and Stoica (1989), Pintelon and Schoukens (2012)

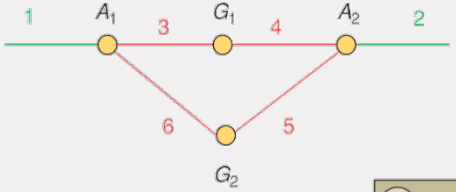
Network models

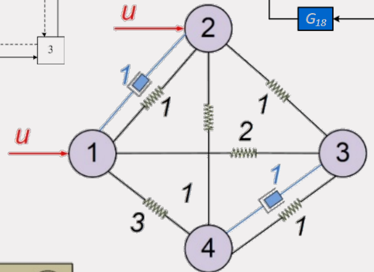


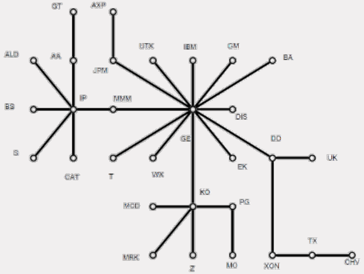


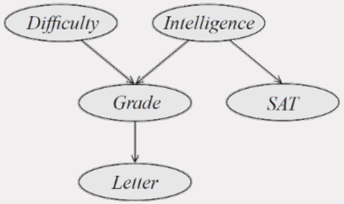


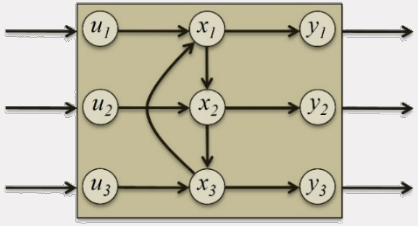


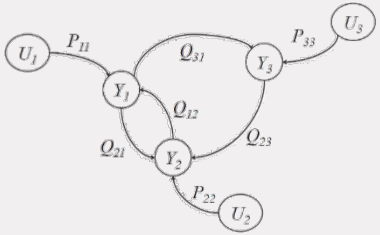












D. Materassi and M.V. Salapaka (2012)

www.momo.cs.okayama-u.ac.jp

E.A. Carara and F.G. Moraes (2008)

P.M.J. Van den Hof et al (2013)

J.C. Willems (2007)

X.Cheng (2019)

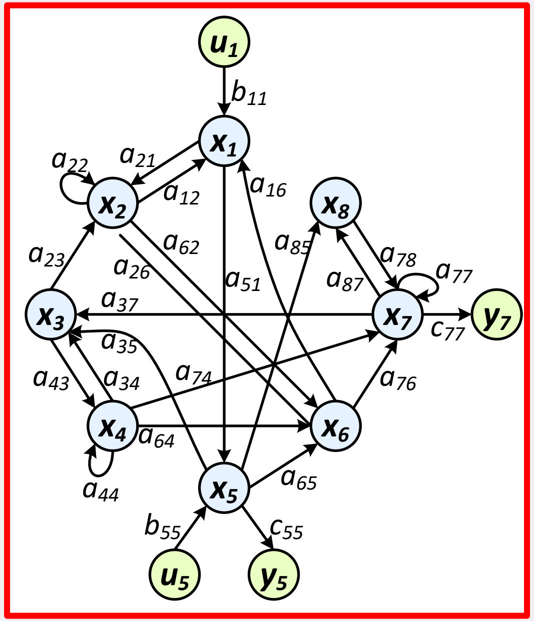
R.N. Mantegna (1999)

D. Koller and N. Friedman (2009)

P.E. Paré et al (2013)

E. Yeung et al (2010)

Network models

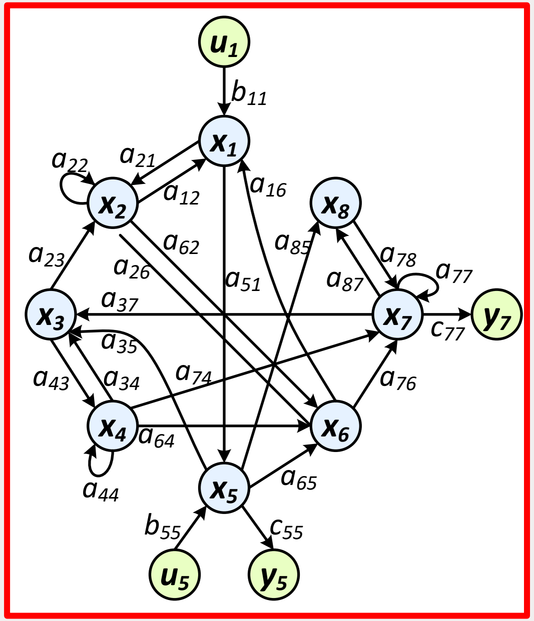


State space representation

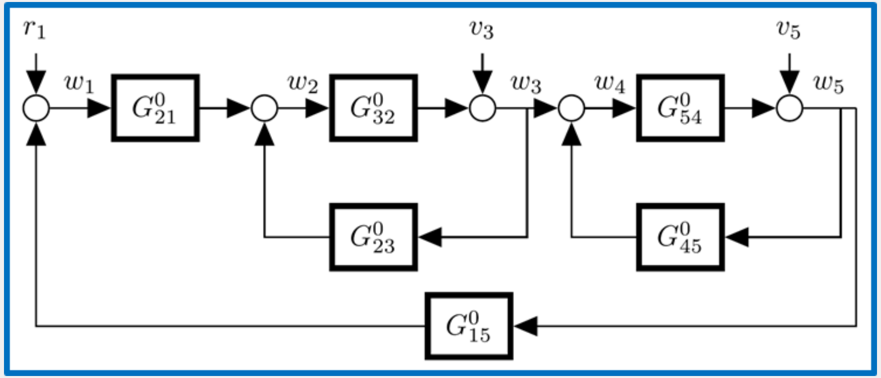
$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}$$

- States as **nodes** in a (directed graph)
- State transitions (1 step in time) reflected by a_{ij}
- Transitions are encoded in **links**
- Effect of transitions are summed in the nodes
- Self loops are allowed
- Actuation (u) and sensing (y) reflected by separate links

Network models



State space representation [1]

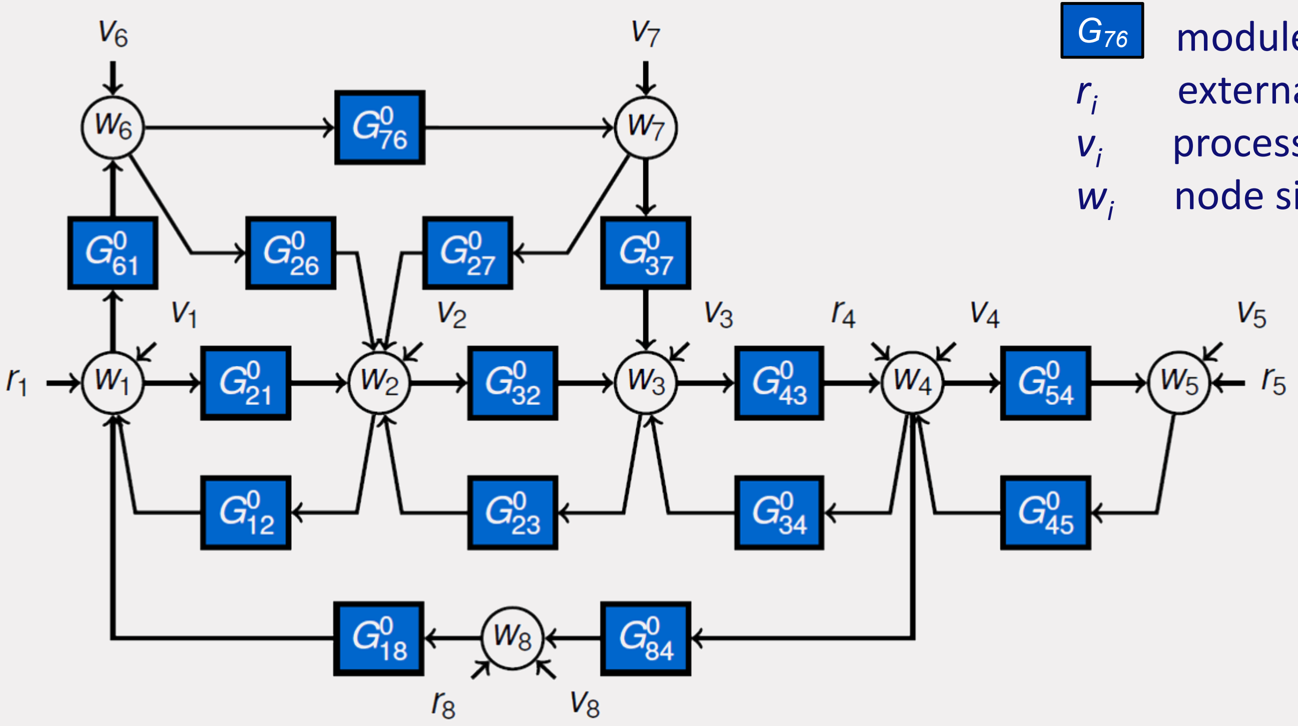


Module representation [2]

Compare e.g. classical signal flow graphs [3]

[1] Goncalves, Warnick, Sandberg, Yeung, Yuan, Scherpen,…
[2] VdH, Dankers, Goncalves, Warnick, Gevers, Bazanella, Hendrickx, Materassi, Weerts,…
[3] S.J. Mason, 1953, 1955.

Dynamic network setup



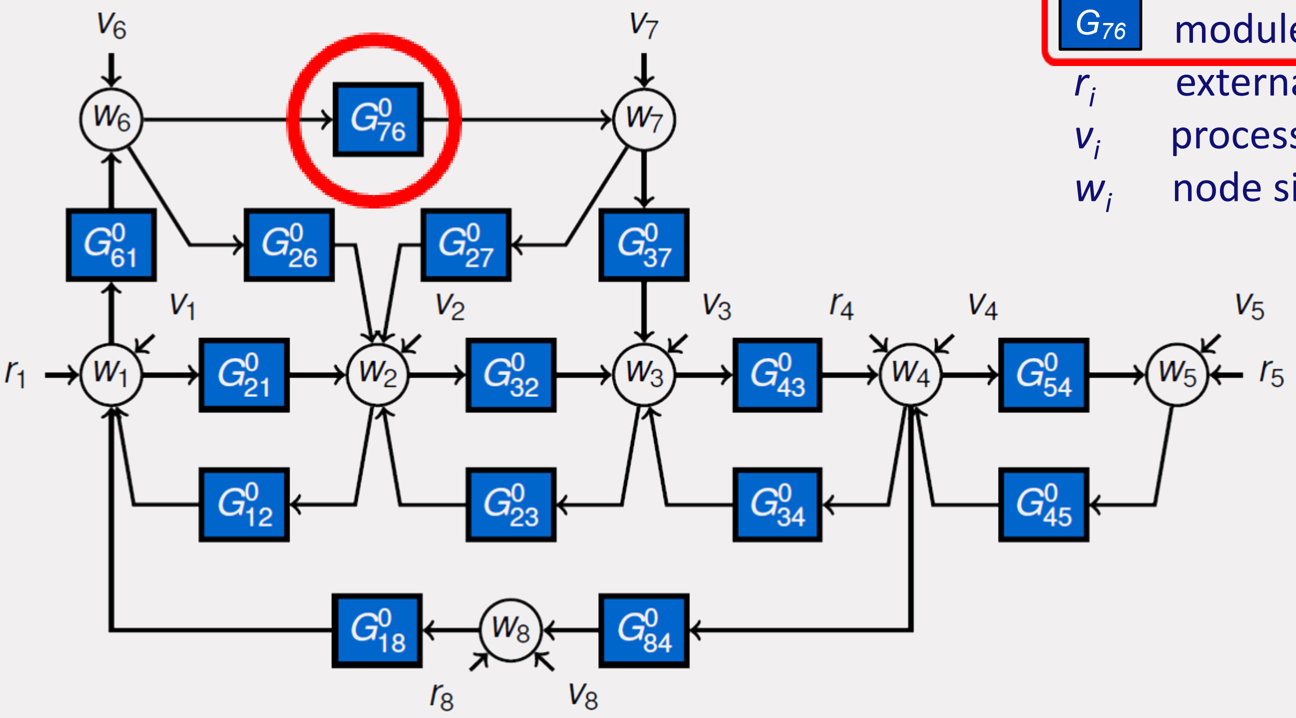
G_{76} module

r_i external excitation

v_i process noise

w_i node signal

Dynamic network setup



G_{76} module
r_i external excitation
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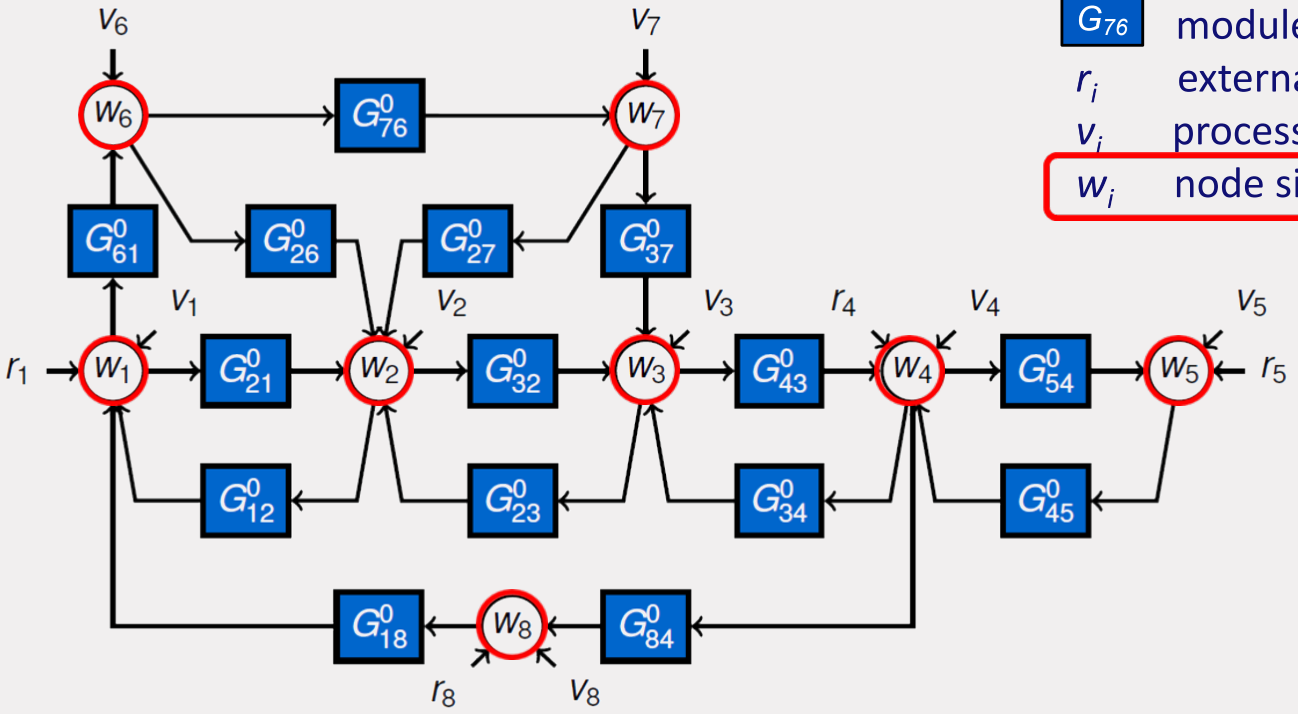
Dynamic network setup



G_{76} module
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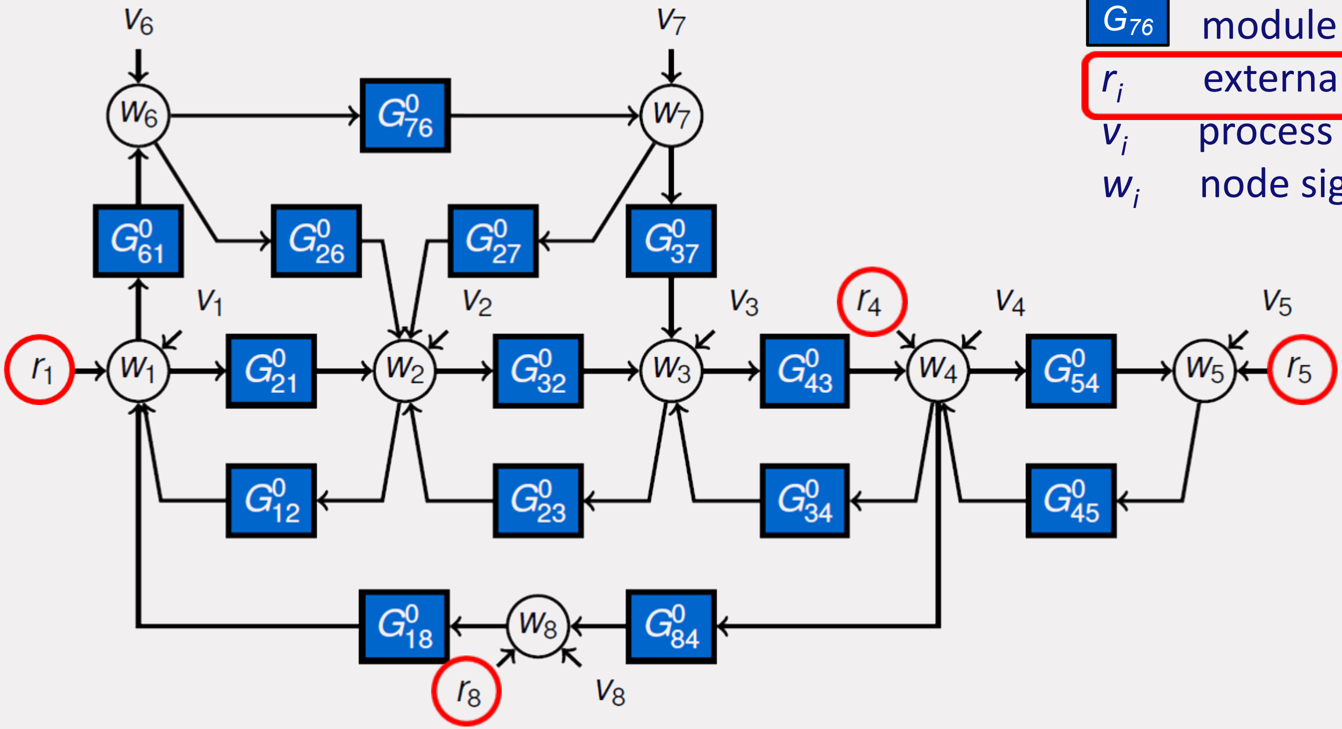
Dynamic network setup



G_{76} module

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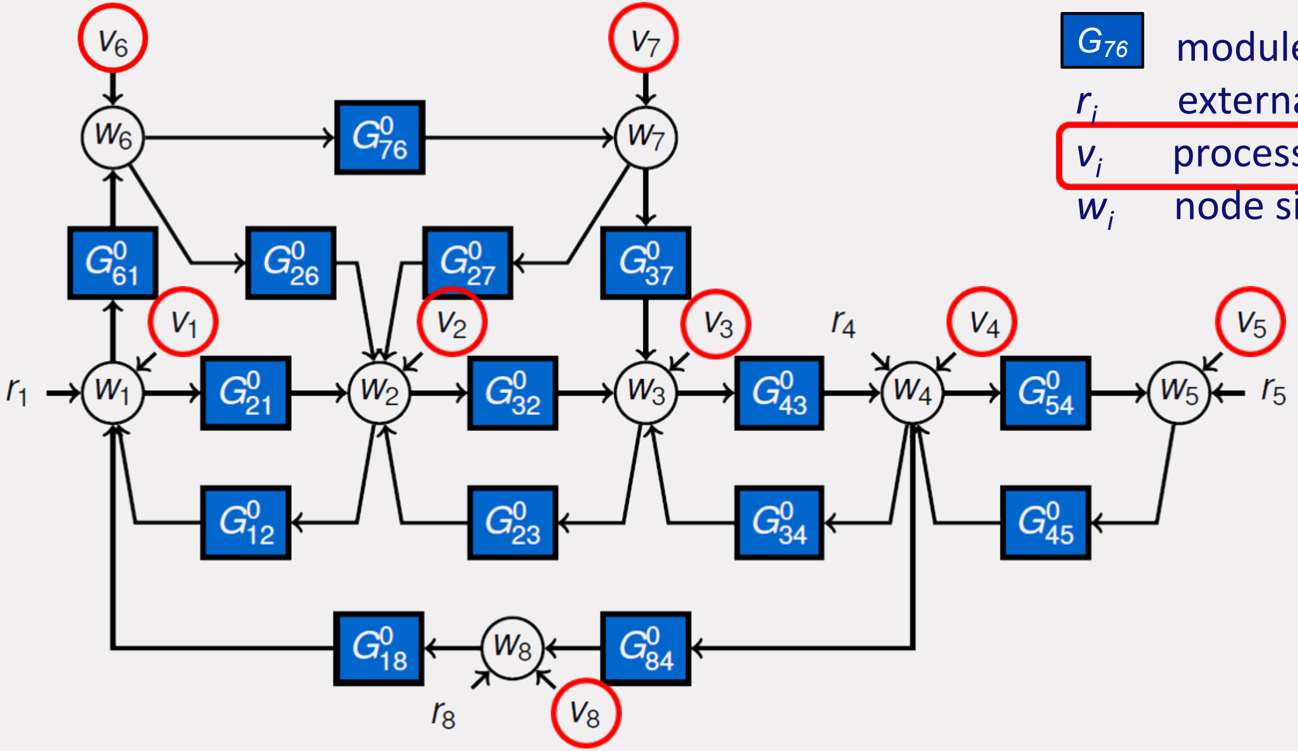
Dynamic network setup



G_{76} module
r_i external excitation
v_i process noise
w_i node signal

Dynamic network setup

Basic building block:

$$w_j(t) = \sum_{k \in \mathcal{N}_j} G^0_{jk}(q) w_k(t) + r_j(t) + v_j(t)$$

w_j: node signal
r_j : external excitation signal
v_j : (unmeasured) disturbance, stationary stochastic process
G^0_{jk}: module, rational proper transfer function

Node signals: $w_1, \cdots w_L$
Interconnection structure / topology of the network is encoded in $\mathcal{N}_j,\ j = 1, \cdots L$

J. Gonçalves and S. Warnick, IEEE TAC, 2008.
PVdH et al., Automatica, 2013.

Dynamic network setup

Collecting all equations:

$$\begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_L(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12}(q) & \cdots & G^0_{1L}(q) \\ G^0_{21}(q) & 0 & \ddots & G^0_{2L}(q) \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1}(q) & G^0_{L2}(q) & \cdots & 0 \end{bmatrix}}_{\text{Network matrix } G^0(q)} \begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_L(t) \end{bmatrix} + R^0 \begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_K(t) \end{bmatrix} + H^0(q) \begin{bmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_p(t) \end{bmatrix}$$

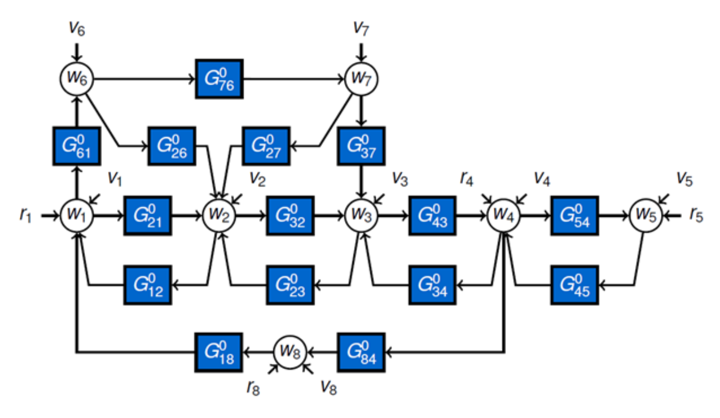
$$w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t); \qquad v(t) = H^0(q)e(t); \quad cov(e) = \Lambda$$

- Typically R^0 is just a (static) selection matrix, indicating which nodes have an excitation signal.
- The topology of the network is encoded in the structure (non-zero entries) of G^0.
- r and e are called external signals.

J. Gonçalves and S. Warnick, IEEE TAC, 2008.
PVdH et al., Automatica, 2013.

Dynamic network setup

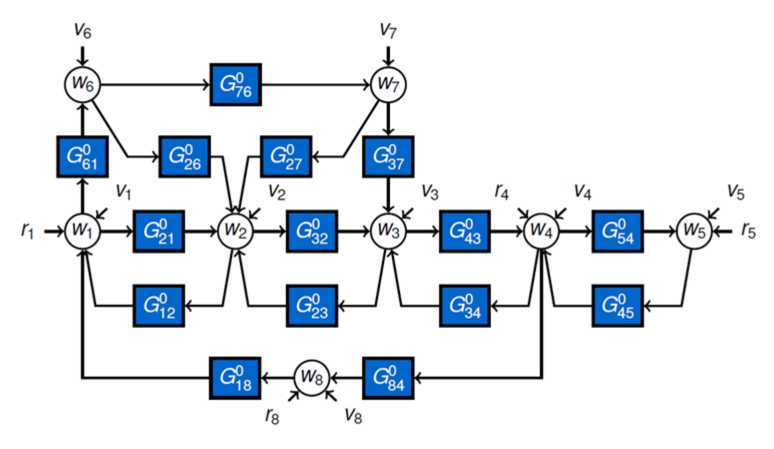
$$w = G^0 w + R^0 r + H^0 e$$



Assumptions:

- Total of L nodes, no self-loops
- Network is well-posed and stable, i.e. $(I - G^0)^{-1}$ exists and is stable
- Modules are dynamic, LTI, proper, may be unstable
- Disturbances can be correlated: H^0 not necessarily diagonal

Data-driven modeling

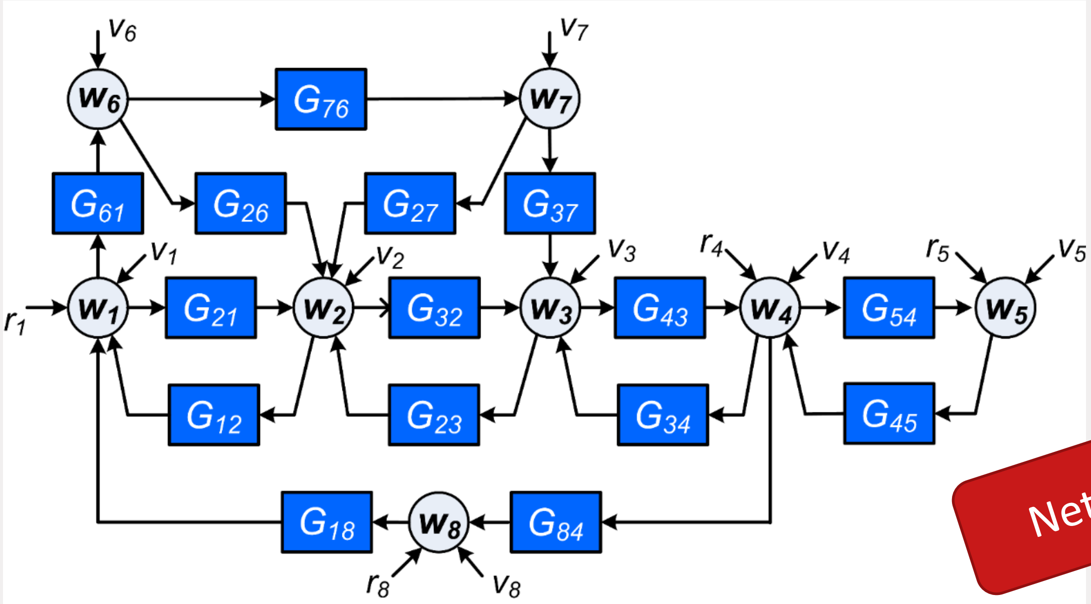


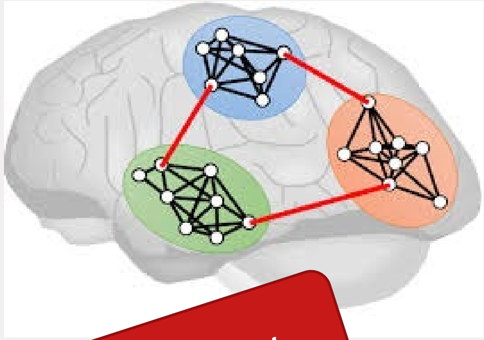
Many new data-driven modeling questions can be formulated

Measured time series:

$$\{w_i(t)\}_{i=1,\cdots L};\ \ \{r_j(t)\}_{j=1,\cdots K}$$

Model learning problems

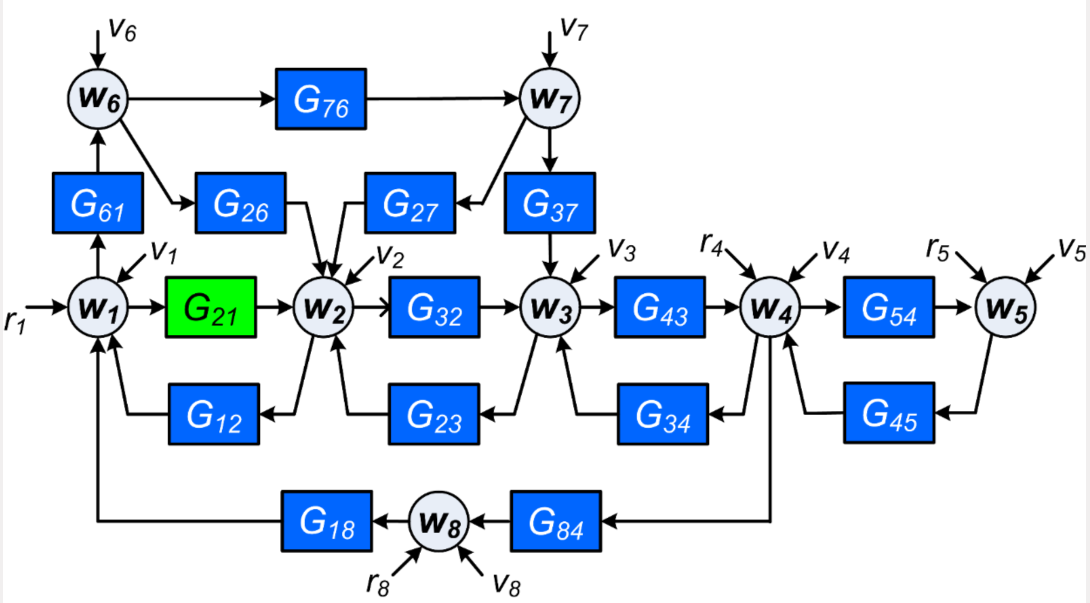




Network identifiability

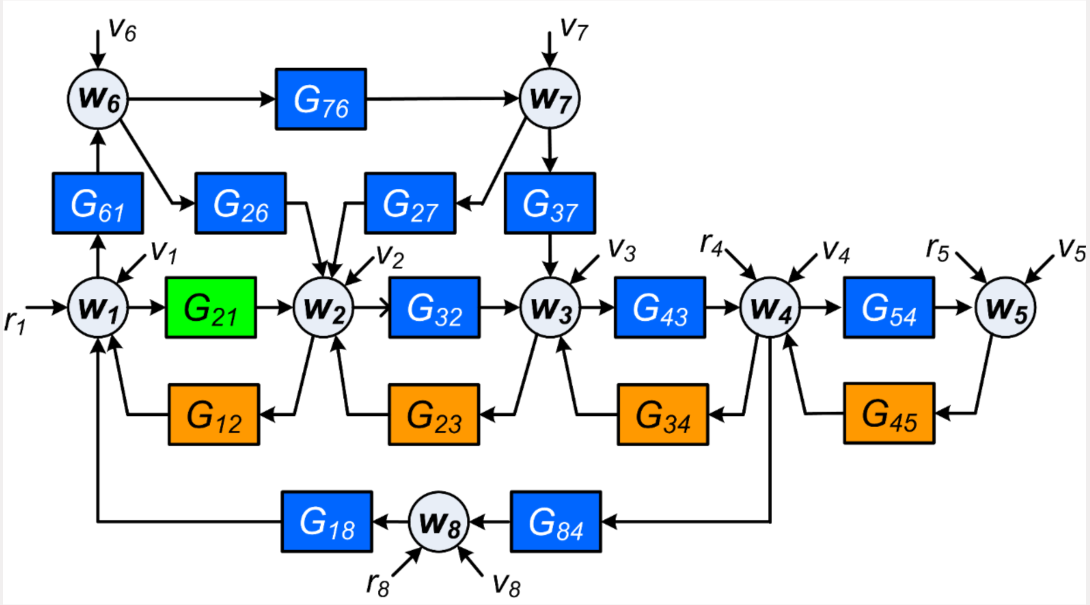
Under which conditions can we estimate from *(w,r)* the topology and/or dynamics of the full network?

Model learning problems



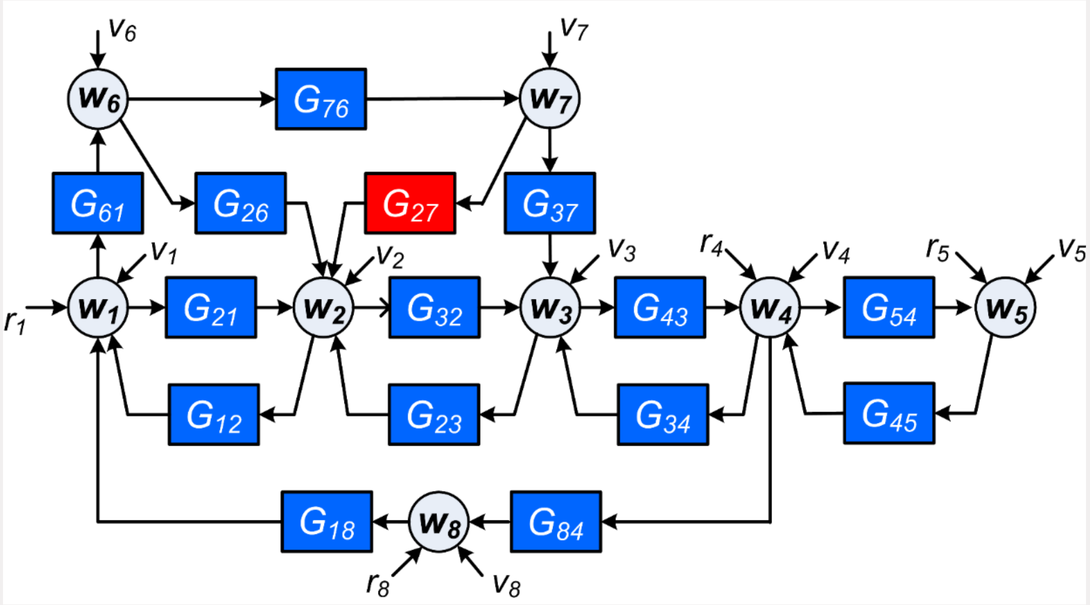
How/when can we learn a local module from data
(with known/unkown network topology) ? Where to sense / actuate?

Model learning problems



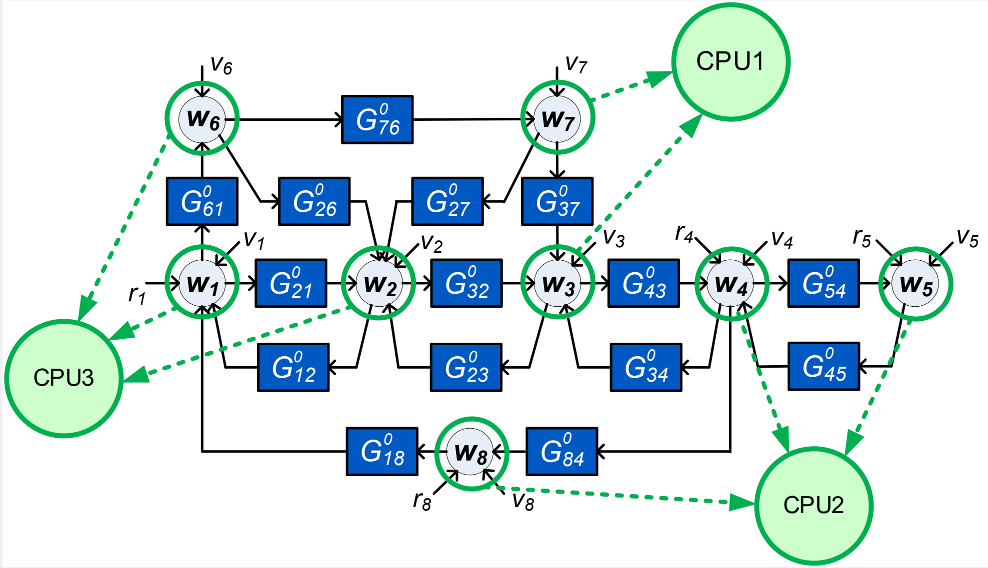
How can we benefit from a priori known modules?

Model learning problems



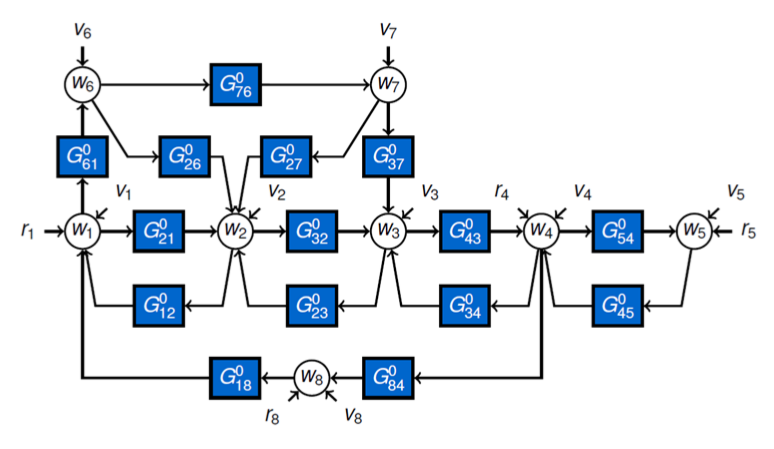
Fault detection and diagnosis; detect/handle nonlinear elements

Model learning problems



Can we distribute the computations?

Dynamic network setup



Measured time series:

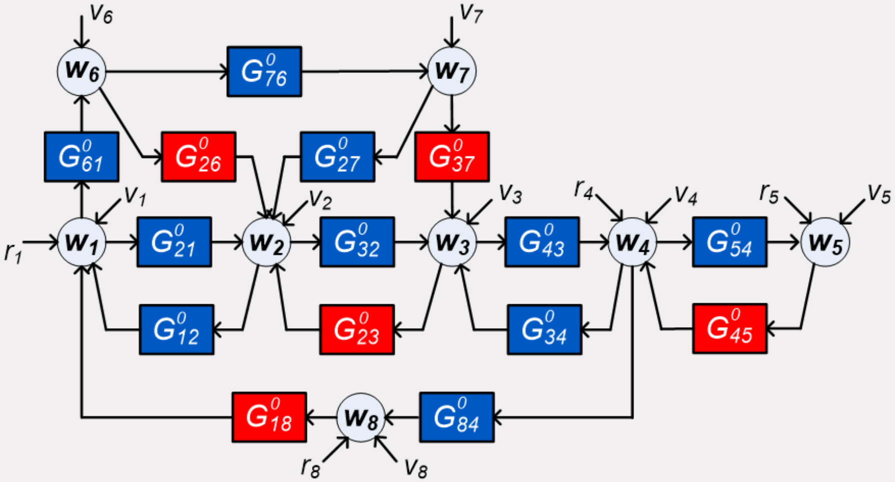
$$\{w_i(t)\}_{i=1,\cdots L};\ \{r_j(t)\}_{j=1,\cdots K}$$

Many new data-driven modeling questions can be formulated

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Fault detection
- User prior knowledge of modules
- Distributed identification
- **Scalable algorithms**

Identifiability

Network identifiability

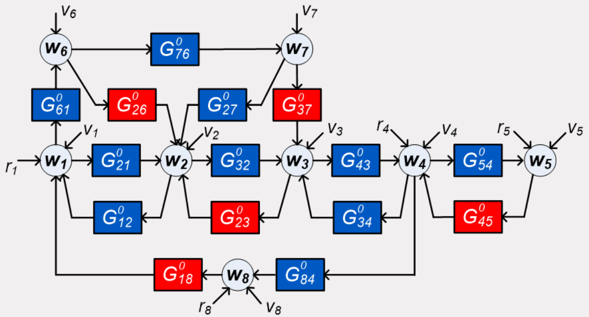


blue = unknown
red = known

Question: Can different dynamic networks be *distinguished* from each other from measured signals w, r ?

OR: If different networks in our model set generate the same w for a given r then we have lack of network identifiability

Network identifiability



The identifiability problem:

The network **model**:

$$w(t) = G(q)w(t) + R(q)r(t) + \underbrace{H(q)e(t)}_{v(t)}$$

can be transformed with any rational $P(q)$:

$$P(q)w(t) = P(q)\{G(q)w(t)+R(q)r(t)+H(q)e(t)\}$$

to an **equivalent model**:

$$w(t) = \tilde{G}(q)w(t) + \tilde{R}(q)r(t) + \tilde{H}(q)e(t)$$

→ **Nonuniqueness,** unless there are structural constraints on G, R, H.

[1] Weerts, Linder et al., Automatica, 2020.
[2] Bottegal et al., SYSID 2017

TU/e

Network identifiability

Consider a **network model set**:

$$\mathcal{M} = \{(G(\theta), R(\theta), H(\theta))\}_{\theta \in \Theta}$$

representing structural constraints on the considered models:

- modules that are fixed and/or zero (topology)
- locations of excitation signals
- disturbance correlation

When are network models **equivalent** in this set?

If they provide the same $T_{wr} := (I-G)^{-1}R,$ and

$$\Phi_{\bar{v}} := (I-G)^{-1}HH^*(I-G)^{-*}$$

with $w(t) = T_{wr}(q)r(t) + \tilde{v}(t)$

[1] Weerts et al., SYSID2015; Weerts et al., Automatica, March 2018;

Network identifiability

Definition Network identifiability[1]

For a network model set $\mathcal{M}$, consider a model $M(\theta_0) \in \mathcal{M}$ and the implication

$$M(\theta_0) \sim M(\theta_1) \implies M(\theta_0) = M(\theta_1),$$
$$\text{for all } M(\theta_1) \in \mathcal{M}$$

Then $\mathcal{M}$ is

- globally identifiable from (w, r) at $M(\theta_0)$ if the implication holds for $M(\theta_0)$;
- globally identifiable from (w, r) if it holds for all $M(\theta_0) \in \mathcal{M}$;
- generically identifiable[2] from (w, r) if it holds for almost all $M(\theta_0) \in \mathcal{M}$;

[1] Weerts et al., Automatica, March 2018;
[2] Hendrickx et al., IEEE-TAC, 2019.

Second network identifiability result

Sufficient condition for network identifiability[1] – general case

Consider model set $\mathcal{M}$, and define for each $j \in [1, L]$:

$\breve{T}_j$:= the transfer function from

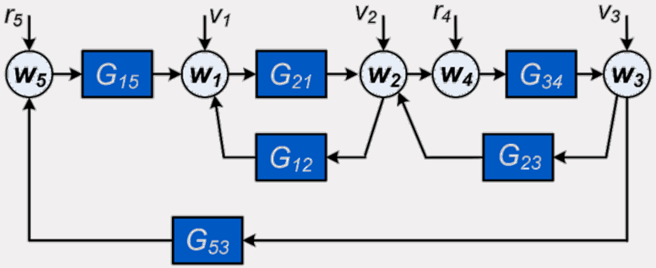
- all external signals (r, e) that do not enter w_j through a parametrized module, to
- all node signals w that map to w_j through a parametrized module.

Then $\mathcal{M}$ is globally network identifiable from (r, w) if for all $j \in [1, L]$:

$$\breve{T}_j \text{ is full row rank for all } \theta \in \Theta.$$

[1] Weerts et al, Automatica, March 2018.

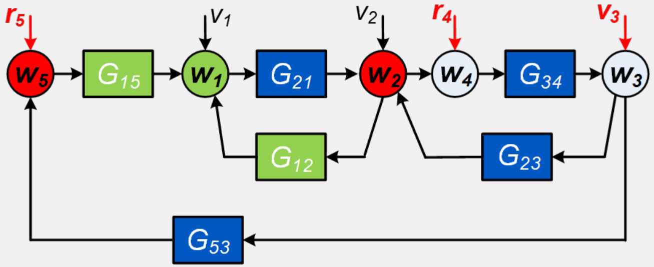
Example 5-node network



Consider the model set determined by:

$$G(\theta) = \begin{bmatrix} 0 & G_{12}(\theta) & 0 & 0 & G_{15}(\theta) \\ G_{21}(\theta) & 0 & G_{23}(\theta) & 0 & 0 \\ 0 & 0 & 0 & G_{34}(\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & G_{53}(\theta) & 0 & 0 \end{bmatrix} \qquad [H \;\; R] = \begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) & 0 & 0 & 0 \\ H_{21}(\theta) & H_{22}(\theta) & 0 & 0 & 0 \\ 0 & 0 & H_3(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Example 5-node network (continued)



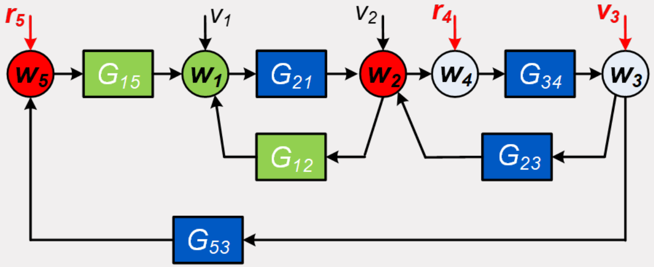
Rank condition:

evaluation of $\breve{T}_j$ for $j = 1$:

$$G(\theta) = \begin{bmatrix} 0 & G_{12}(\theta) & 0 & 0 & G_{15}(\theta) \\ G_{21}(\theta) & 0 & G_{23}(\theta) & 0 & 0 \\ 0 & 0 & 0 & G_{34}(\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & G_{53}(\theta) & 0 & 0 \end{bmatrix} \qquad [H \;\; R] = \underbrace{\begin{bmatrix} H_{11}(\theta) & H_{12}(\theta) & 0 & 0 & 0 \\ H_{21}(\theta) & H_{22}(\theta) & 0 & 0 & 0 \\ 0 & 0 & H_3(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{K+p=5}$$

$$\breve{T}_1 : \begin{bmatrix} v_3 \\ r_4 \\ r_5 \end{bmatrix} \rightarrow \begin{bmatrix} w_2 \\ w_5 \end{bmatrix}$$ has to have full row rank $\forall \theta \in \Theta$

Example 5-node network (continued)



Issues:

- Such a rank test is not easy to apply
- and needs to be done for every $j = 1, \cdots L$

Generic identifiability provides more attractive and constructive conditions

(see next presentation by Xiaodong Cheng)

Summary network modeling

- Introduced an estimation-oriented way for modelling dynamic networks
- Extended transfer function approach approach to include structure (topology)
- This raises an abundance of new data-driven modeling questions
- Introduced the concept of network identifiability

Tutorial session: Data-driven modeling in dynamic networks

- Introduction (Paul Van den Hof) 15:30 – 16:00
    - Modeling framework, identification challenges, identifiability

- Graph-based method for analysing identifiability and allocating excitation signals (Xiaodong Cheng) 16:00 – 16:30

- Algorithm for full network identification and a case study in gas pipeline monitoring (Arne Dankers) 16:30 – 17:00

- Identification of single modules in a dynamic network (Karthik Ramaswamy) 17:00 – 17:30

Feel free to raise questions in the Q&A

Further reading

- P.M.J. Van den Hof, A. Dankers, P. Heuberger and X. Bombois (2013). Identification of dynamic models in complex networks with prediction error methods - basic methods for consistent module estimates. *Automatica*, Vol. 49, no. 10, pp. 2994-3006.
- A. Dankers, P.M.J. Van den Hof, X. Bombois and P.S.C. Heuberger (2015). Errors-in-variables identification in dynamic networks - consistency results for an instrumental variable approach. *Automatica*, Vol. 62, pp. 39-50, December 2015.
- A. Dankers, P.M.J. Van den Hof, P.S.C. Heuberger and X. Bombois (2016). Identification of dynamic models in complex networks with predictior error methods - predictor input selection. *IEEE Trans. Autom. Contr.*, *61 (4),* pp. 937-952, 2016.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2018). Identifiability of linear dynamic networks. *Automatica*, 89, pp. 247-258, March 2018.
- H.H.M. Weerts, P.M.J. Van den Hof and A.G. Dankers (2018). Prediction error identification of linear dynamic networks with rank-reduced noise. *Automatica, 98,* pp. 256-268, December 2018.
- H.H.M. Weerts, J. Linder, M. Enqvist and P.M.J. Van den Hof (2019). Abstractions of linear dynamic networks for input selection in local module identification. Automatica, Vol. 117, July 2020.
- P.M.J. Van den Hof, A.G. Dankers and H.H.M. Weerts (2018). System identification in dynamic networks. Computers & Chemical Engineering, Vol. 109, pp. 23-29, January 2018.

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