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Path-based data-informativity conditions for single module identification in dynamic networks

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Dynamic network setup

$$w_j(t) = \sum_{k \in \mathcal{N}_j} G_{ji}(q) w_i(t) + r_j(t) + v_j(t)$$

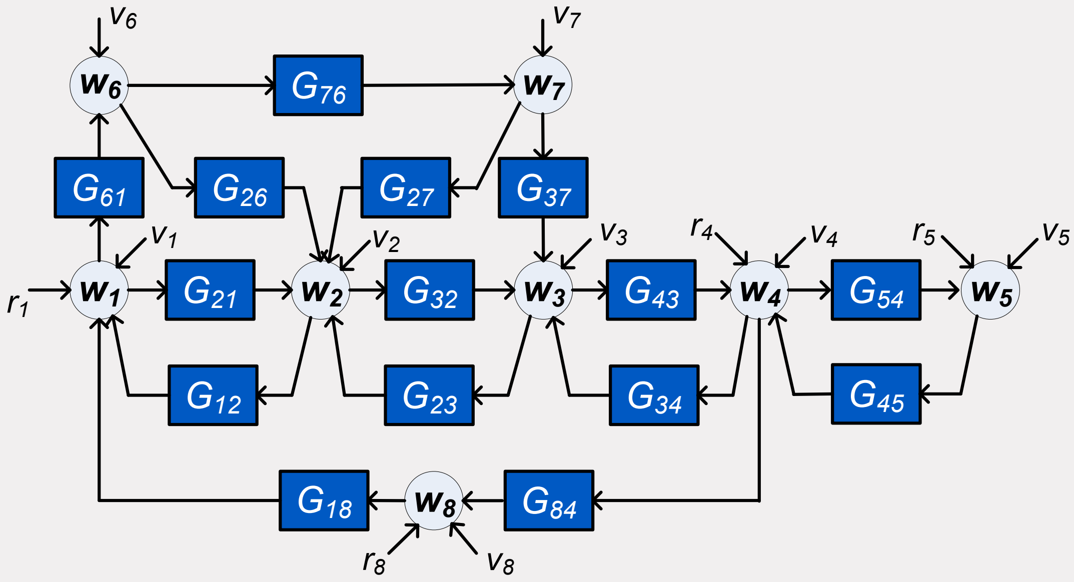
w_j node signals, $j = 1, \cdots L$

G_{ji} modules (LTI)

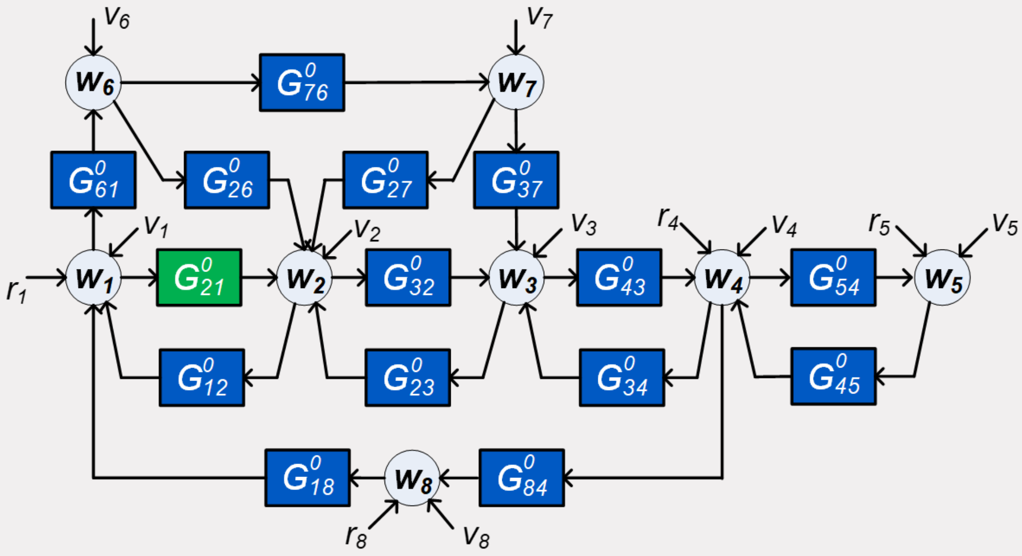
r_j external excitation

v_j disturbance signal

$$w = Gw + Rr + He$$



Single module identification



The problem:

For a network with known topology:

Identify G^0_{21} on the basis of selected measured signals (w, r)

Preference for "local" measurements and limited excitation

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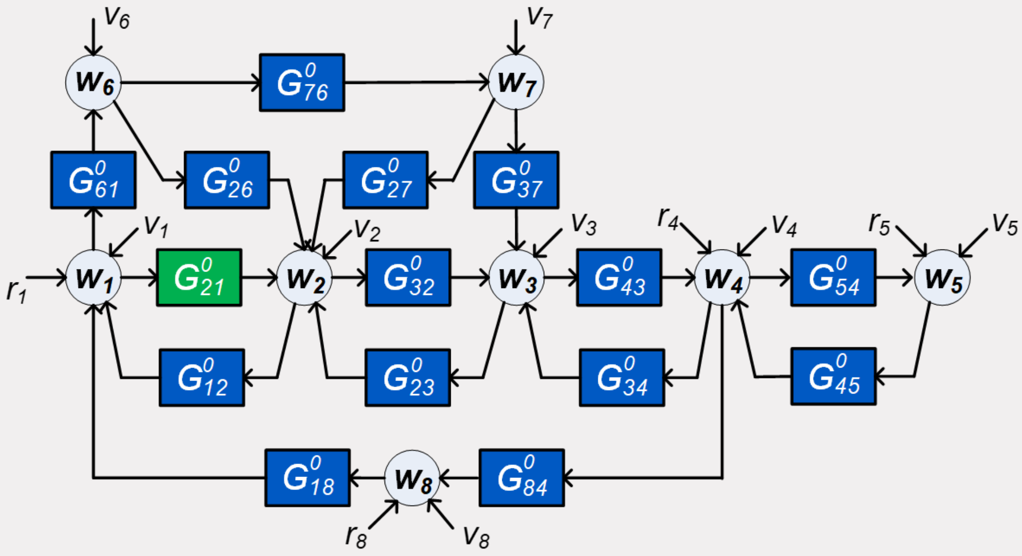
Single module identification



Options:

- Indirect method[1],[2]
  Dependent on excitation from $\boldsymbol{r}$ only

- Direct method[1],[3],[4]
  Excitation from $(\boldsymbol{r}, \boldsymbol{v})$
  Maximum likelihood results

[1] VdHof et al., Automatica 2013
[2] Gevers et al., SYSID 2018; Bazanella et al., CDC 2019
[3] Dankers et al., IEEE Trans. Autom Control, 2016
[4] Ramaswamy et al., IEEE Trans. Autom Control, 2021

Direct method

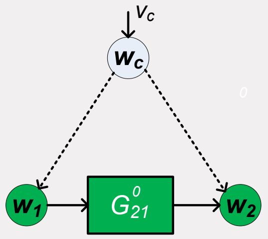
Full network equation: $$w = Gw + Rr + He$$

Signal selection

Predictor model after signal selection: $$w_{\mathcal{Y}} = \bar{G}w_{\mathcal{D}} + \bar{R}r_{\mathcal{P}} + \bar{H}\xi_{\mathcal{Y}}$$

This can be handled in a direct-type identification method if

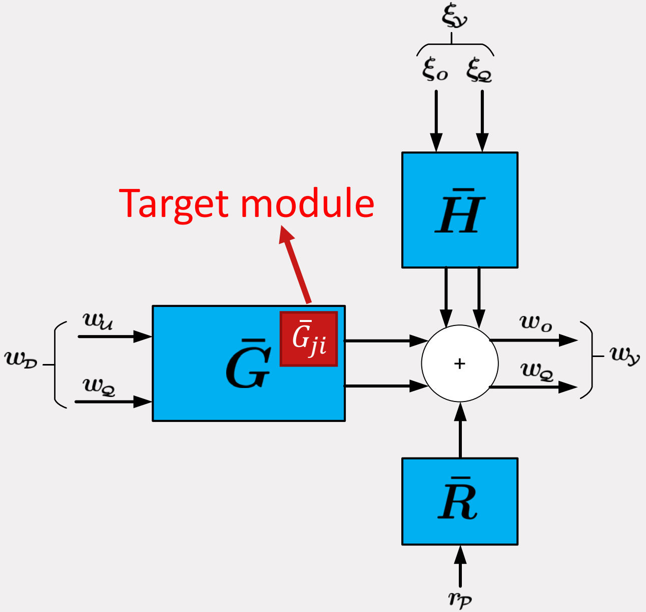
1. $\bar{G}_{ji} = G_{ji}$ for the target module
2. $\bar{R}$ is a selection matrix
3. There are no confounding variables that affect the estimation of $\bar{G}_{ji}$

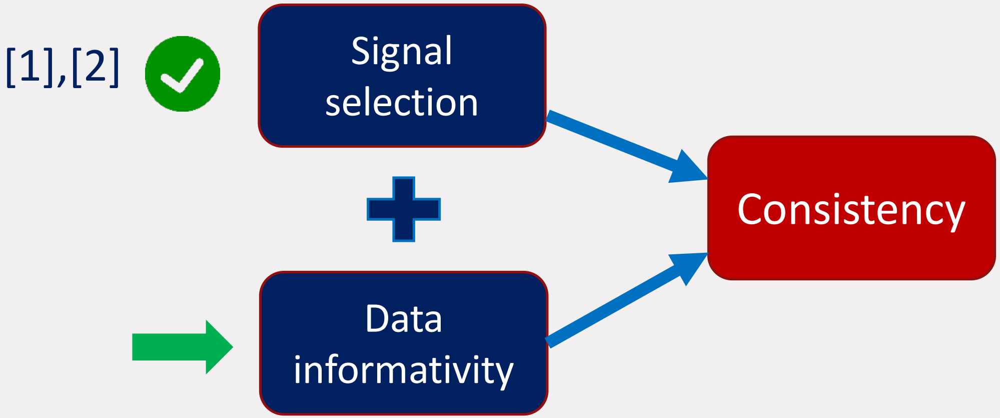


This can lead to a MIMO predictor model.

Direct method

Resulting predictor model:





There can be common signals in input and output: $w_{\mathcal{Q}}$

[1] Dankers et al., IEEE Trans. Autom Control, 2016
[2] Ramaswamy et al., IEEE Trans. Autom Control, 2021

Data informativity (classical definition)

Predictor model: $\hat{w}_{\mathcal{Y}}(t,\theta) = W(q,\theta)z(t)$

with $z(t) := \begin{bmatrix} w_{\mathcal{Y}}(t) \\ w_{\mathcal{D}}(t) \\ r_{\mathcal{P}}(t) \end{bmatrix}$ for a model set $\mathcal{M}$ parametrized by $\theta \in \Theta$

Then a data sequence $\{z(t)\}_{t=0,\cdots}$ is informative with respect to $\mathcal{M}$ if for any two models in $\mathcal{M}$:

$$\boxed{\bar{\mathbb{E}}[(W_1(q)-W_2(q))z(t)]^2 = 0 \implies W_1(e^{i\omega}) \equiv W_2(e^{i\omega})}$$

A sufficient condition for this is that z is persistently exciting:

$$\boxed{\Phi_z(\omega) > 0 \text{ for almost all } \omega}$$

[1] L. Ljung, Englewood Cliffs, NJ: Prentice-Hall, 1999

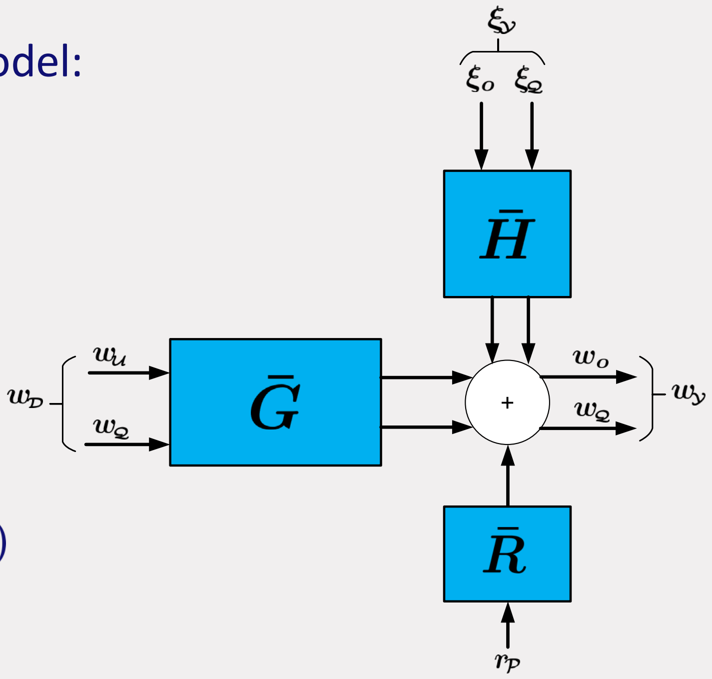
Spectrum condition - network case

In the situation of our specific predictor model:

$$\boxed{\Phi_\kappa(\omega) > 0 \text{ for almost all } \omega}$$

$$\kappa(t) := \begin{bmatrix} w_{\mathcal{D}}(t) \\ \xi_{\mathcal{Y}}(t) \end{bmatrix}$$

- Note that κ is a filtered version of (r, e)
- with (r, e) persistently exciting

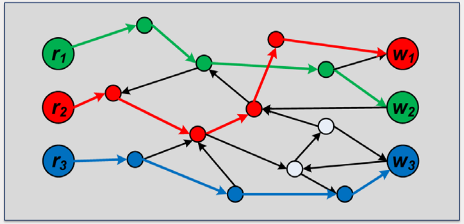


[1] K.R. Ramaswamy and P.M.J. Van den Hof., IEEE-TAC, 2021.

Data informativity (path-based condition)

By exploiting the fact that

- $$\kappa(t) = F(q)\begin{bmatrix} r(t) \\ e(t) \end{bmatrix}$$
- p.e. of κ is determined by the row rank of F
- which can be evaluated generically [1],[2] by the number of **vertex disjoint paths** between inputs and outputs



$$b_{\mathcal{R}\to\mathcal{W}} = 3$$

we arrive at the **final result**:

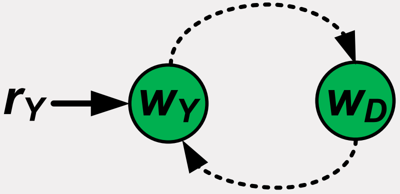
Persistence of excitation of κ holds generically
if there are
$|\mathcal{D}|$ vertex disjoint paths between external signals $(r_{\mathcal{P}}, x_{\mathcal{U}})$ and $w_{\mathcal{D}}$

[1] Van der Woude, 1991
[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019.

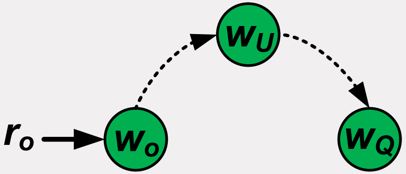
External signals on original network

Generic data informativity check becomes:
$|\mathcal{D}|$ vertex disjoint paths between external signals $(r_\mathcal{P}, x_\mathcal{U})$ and $w_\mathcal{D}$

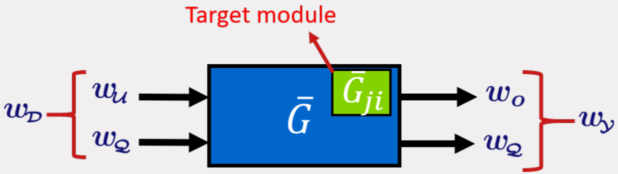
Signals in $r_\mathcal{P} \in r_\mathcal{Y}$



Loops through $w_\mathcal{Y}$
pass through $w_\mathcal{D}$



Path from w_o to $w_\mathcal{Q}$
passes through $w_\mathcal{U}$



Signals in $x_\mathcal{U}$: All external signals (r, e) that have a direct or unmeasured path to $w_\mathcal{U}$

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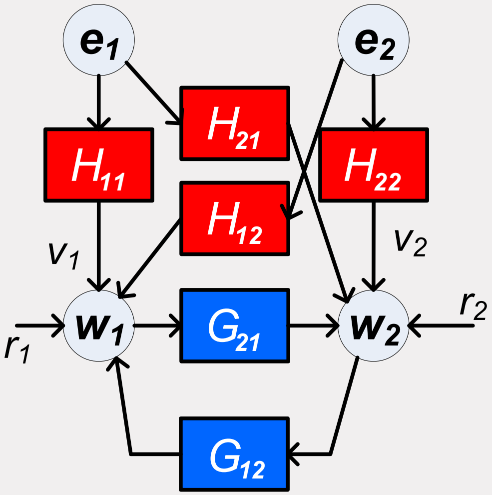
Example

Target: identify G_{21}

Predictor model: $\underbrace{\{w_1\}}_{w_{\mathcal{D}}} \rightarrow \underbrace{\{w_1, w_2\}}_{w_{\mathcal{Y}}}$

2 x 2 noise model accounts for confounding variable

$w_{\mathcal{Q}} = \{w_1\} \quad w_{\mathcal{U}} = \emptyset \quad x_{\mathcal{U}} = \emptyset$



Result 1:

Neither r_1 nor r_2 can contribute to $r_{\mathcal{P}}$ ⟶ data informativity condition is not satisfied

Result 2: Change predictor model to: $\underbrace{\{w_1, w_2\}}_{w_{\mathcal{D}}} \rightarrow \underbrace{\{w_1, w_2\}}_{w_{\mathcal{Y}}}$

Both r_1 and r_2 contribute to $r_{\mathcal{P}}$ ⟶ data informativity condition is satisfied

Conclusion

- Data-informativity condition specified for direct identification of single modules in dynamic networks, with given topology
- For satisfying data-informativity generically, path-based conditions can be formulated on the network graph
- Together with the path-based conditions for appropriate predictor model selection, a full set of path-based conditions is formulated for consistent estimation of a single module
- To be used also for external signal allocation (experimental setup)

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