Single module identification in dynamic networks – the current status

Paul Van den Hof, Karthik Ramaswamy

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Open invited track: “Data-driven modeling and learning in dynamic networks”

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Introduction – dynamic networks

Decentralized process control

Smart power grid

Stock market

PCB testing

Autonomous driving

Brain network

Hydrocarbon reservoirs

Physiological models

www.envidia.com

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P. Hagmann et al. (2008)

Mansoori (2014)

Christie, Achenie and Oggunnaike (2014)

Materassi and Innocenti, 2010

T&M Solutions, Romex BV

Betterworldsolutions.eu

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Christie, Achenie and Oggunnaike (2014)
Dynamic network setup

- $G_{76}$ module
- $w_i$ node signal
- $r_i$ external excitation
- $v_i$ process noise
Dynamic network setup

- $G_{76}$ module
- $w_i$ node signal
- $r_i$ external excitation
- $v_i$ process noise
Dynamic network setup

- \( v_i \): node signal
- \( r_i \): external excitation
- \( v_i \): process noise
Dynamic network setup

- $v_i$: process noise
- $w_i$: node signal
- $r_i$: external excitation
- $G_{ij}$: module
Dynamic network setup

$G_{76}$ module

$w_i$ node signal

$r_i$ external excitation

$v_i$ process noise
Dynamic network setup

Assumptions:

- Total of \( L \) nodes
- Network is well-posed and stable
- Modules are dynamic LTI, may be unstable
- Disturbances are stationary stochastic and can be correlated

\[
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L 
\end{bmatrix} = 
\begin{bmatrix}
  0 & G_{12}^0 & \cdots & G_{1L}^0 \\
  G_{21}^0 & 0 & \cdots & G_{2L}^0 \\
  \vdots & \vdots & \ddots & \vdots \\
  G_{L1}^0 & G_{L2}^0 & \cdots & 0 
\end{bmatrix} 
\begin{bmatrix}
  w_1 \\
  w_2 \\
  \vdots \\
  w_L 
\end{bmatrix} + R^0 
\begin{bmatrix}
  r_1 \\
  r_2 \\
  \vdots \\
  r_K 
\end{bmatrix} + 
\begin{bmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_L 
\end{bmatrix}
\]

\( w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t) \)

\( v(t) = H^0(q)e(t) \)

Many data-driven modeling questions can be formulated

- Identification of a local module (known topology)
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Fault detection
- User prior knowledge of modules
- Distributed identification
- Scalable algorithms

Measured time series:
\[ \{w_i(t)\}_{i=1,\ldots,L}; \quad \{r_j(t)\}_{j=1,\ldots,K} \]
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• Introduction and network model
• Single module identification: what’s the problem?
• Indirect methods
• Direct methods
• Algorithmic aspects
• Single module identifiability
• Conclusions
Single module identification

The problem:
For a network with known topology:
Identify $G_{21}^0$ on the basis of selected measured signals $(w, r)$

Preference for “local” measurements and limited excitation
Single module identification

Naïve approaches:
• identify based on $w_2$ and $w_1$; or
• identify based on $T_{w_2r_1}T_{w_1r_1}^{-1}$ do not work,
e.g. because of parallel paths
Naïve approaches:
• identify based on $w_2$ and $w_1$; or
• identify based on $T_{w_2r_1}T_{w_1r_1}^{-1}$
do not work,
e.g. because of parallel paths
Single module identification

Approaches to the problem:

1. **Prediction error methods**
   VdH et al. (2013); Dankers et al. (2015, 2016); Galrinho et al. (2017); Everitt et al. (2018); Gevers et al. (2018); Bazanella et al. (2017, 2019), Hendrickx et al. (2019), Ramaswamy et al. (2018, 2019, 2020);
   generalizations of closed-loop methods, requiring choice of predictor model

2. **Alternatives**
   • Non-parametric methods, based on Wiener filters and d-separation
     Materassi & Salapaka, (2015,2020)
   • Subspace methods
     Yu and Verhaegen, TAC (2018)
Single module identification

Prediction error methods:

**Choice of predictor model**, leading to prediction errors:

Direct method: \[ \varepsilon(t, \theta) = \tilde{H}(q, \theta)^{-1}[w_Y(t) - \tilde{G}(q, \theta)w_D(t)] \]

direct estimation of target module

Indirect method: \[ \varepsilon(t, \theta) = \tilde{H}(q, \theta)^{-1}[w_Y(t) - \tilde{T}(q, \theta)r_D(t)] \]

indirect estimation through post-processing

Generalized method: \[ \varepsilon(t, \theta) = \tilde{H}(q, \theta)^{-1}[w_Y(t) - \tilde{G}(q, \theta)w_{D,w}(t) - \tilde{T}(q, \theta)r_{D,r}(t)] \]
Single module identification

Prediction error methods:

Main differences:

Direct method: \( \varepsilon(t, \theta) = \tilde{H}(q, \theta)^{-1}[w_Y(t) - \tilde{G}(q, \theta)w_D(t)] \)

Predictor inputs \( w_D(t) \) receive excitation from both \( r \) and \( e \) signals

Indirect method: \( \varepsilon(t, \theta) = \tilde{H}(q, \theta)^{-1}[w_Y(t) - \tilde{T}(q, \theta)r_D(t)] \)

Predictor inputs \( r_D(t) \) receive excitation from \( r \) signals only

Overall: indirect methods have stronger requirements on the presence of measurable external excitation signals \( r \) \( \rightarrow \) more expensive experiments
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Multi-input single-output identification problem to be addressed by a closed-loop identification method
Indirect methods

How to choose predictor inputs and outputs?

- Select output $w_j$ and all its in-neighbors $w_N$ as predictor output; $r_D$ as predictor input.
- Estimate $\hat{T}_{N_r}$ and $\hat{T}_{j_r}$ consistently, and determine:
  $$\hat{G}_{jN} = \hat{T}_{j_r} \hat{T}_{N_r}^{-1}$$ \[1\]
- or through IV or two-stage method\[2\]
- freedom in location of r-signals (e.g. directly on $w_N$)
- dual (outneighbour) setup is also possible\[1\]
- we do not necessarily need all in-neighbors to be included in $w_N$

[1] Gevers et al., SYSID 2018; Hendrickx et al, TAC 2019; Bazanella et al., CDC 2019
[2] VdHof et al., Automatica 2013; Dankers et al., Automatica 2015
Indirect methods

How to choose predictor inputs and outputs?

Selection of signals in $w_Y$:

- Parallel path and loop condition

All parallel paths, and loops around the output, should pass through a signal in $w_Y$

Indirect methods

- Parallel path and loop condition results from theory of immersion\(^1\): removing node signals, while retaining the behaviour of the remaining nodes

With network abstractions\(^2\) this can further be generalized:

Measuring descendants of the requested nodes instead

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\(^1\) Dankers et al., IEEE-TAC, 2016; F. Dörfler and F. Bullo, 2013

\(^2\) Linder and Enqvist, 2017; Weerts, Linder et al., Automatica, July 2020
Indirect methods

• Relatively simple methods for **consistent estimation** of target module

• High requirements on presence of excitation signals \( r \)
  leading to “expensive” experiments

  No use of excitation through disturbance signals

As alternative: **direct method**
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Direct method

- Estimate transfer $w_D \rightarrow w_Y$ and model the disturbance process on the output.
- Consistent estimate and ML properties
- Provided there is enough excitation, through external signals $r$ and $e$
- Input signal set $w_D$ can be further reduced\[1\]

Additional problem:

If:  
• $v$ signals are correlated, i.e. $\Phi_v(\omega)$ non-diagonal, or
  • some in-neighbors of $w_Y$ are not included in $w_D$

Then confounding variables can occur, destroying the consistency results

$\varepsilon(t, \theta) = \bar{H}(q, \theta)^{-1}[w_Y(t) - \bar{G}(q, \theta)w_D(t)]$

[1] Dankers et al., IEEE-TAC, 2016; Dankers et al., IFAC 2017
Direct method

**Confounder variable** \(^{(1)}^{[1]}^{[2]}\): Unmeasured signal that has (unmeasured paths) to both the input and output of an estimation problem.

Can be addressed in two ways\(^{[3]}\):
- by adding an additional node signal to \(w_D\), and blocking an unmeasured path; OR
- by adding the affected signal in \(w_D\) to \(w_Y\) and model the correlated disturbances

Resulting predictor model can become a MIMO model

\(^{[2]}\) A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.
\(^{[3]}\) PVdH et al, CDC 2019; Ramaswamy et al., 2020
Direct method

Example of confounding variable handling:

Non-measurable $w_7$ is a confounding variable

Two possible solutions:

1. Include $w_4$ → add predictor input
   
   \[ w_D = \{ w_1, w_3, w_4, w_6 \} \quad w_Y = \{ w_2 \} \]

2. Predict $w_1$ too → add predictor output
   
   \[ w_D = \{ w_1, w_3, w_6 \} \quad w_Y = \{ w_1, w_2 \} \]

Relation with d-separation in graphs (Materassi & Salapaka)[1]

Direct method - Algorithm for signal selection

For estimating target module $G_{ji}$:

1. Select $w_D = w_i$ and $w_Y = w_j$
2. Add node signals to $w_D$ to satisfy the parallel path and loop condition
3. Extend $w_D$ and/or $w_Y$ so as to avoid confounding variables

Algorithm always reaches a convergence point where conditions are satisfied.

The choice options lead to different end-results for signals to be included
   different predictor models
Direct method

General setup:

Different predictor models:

- Full input case: include all in-neighbors of $w_y$
- Minimum node signals case: maximize number of outputs
- User selection case: dedicated choice based on measurable nodes
Consistency result

Conditions for consistent (and ML) estimation of $G_{ji}$\textsuperscript{[1]}:

- System in the model set,
- Parallel path and loop condition satisfied
- Confounding variables handled appropriately
- Persistence of excitation, i.e. $\Phi_\kappa(\omega) > 0$ at a sufficient number of frequencies, with

$$
\kappa = \begin{bmatrix} w_D \\ \xi_Q \\ w_0 \end{bmatrix}
$$

and $\xi_Q$ the innovation process of $w_Q$

(can also be phrased as path-based condition\textsuperscript{[2]})

- Requirements on signals $r$ increase with increasing number of outputs

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\[1\] K.R. Ramaswamy et al., ArXiv 2019, IEEE-TAC, provis accepted.

\[2\] VdH et al., CDC-2020 submitted
Example - direct method & indirect method

\[ \{w_1, w_3, w_5\} \rightarrow \{w_2\} \]

Direct method [1]

Example - direct method & indirect method

\[ \{w_1, w_3, w_5\} \rightarrow \{w_2\} \]

Direct method \([1]\)

\[ \{r_1, r_3, r_5\} \rightarrow \{w_1, w_2, w_3, w_5\} \]

Indirect method \([2]\)

What can we do if parallel path/loop conditions cannot be satisfied?
What can we do if certain nodes cannot be excited?

We combine the ideas of direct and indirect methods to increase flexibility.

Example - direct method & indirect method

\[
\{w_1, w_3, w_5\} \rightarrow \{w_2\} \\
\text{Direct method}
\]

\[
\{r_1, r_3, r_5\} \rightarrow \{w_1, w_2, w_3, w_5\} \\
\text{Indirect method}
\]
Example - direct method & indirect method

- Include both internal nodes and external excitation as predictor inputs
- Instead of measuring a parallel path we excite it and measure a descendant
- Generalized method increases flexibility in selecting sensors/actuators

\[ \{w_1, w_3, w_5\} \rightarrow \{w_2\} \]

**Direct method**

\[ \{r_1, r_3, r_5\} \rightarrow \{w_1, w_2, w_3, w_5\} \]

**Indirect method**

\[ \{w_1, w_4, r_2, r_3\} \rightarrow \{w_2, w_4\} \]

**Generalized method [1]**

[1] K.R. Ramaswamy et al., CDC 2019
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Machine learning in local module identification

- MISO/MIMO identification with all modules parameterized
- Brings in some major computational complexity
- We need only the target module. No NUISANCE!
Machine learning in local module identification

Maximize marginal likelihood of output data: $\hat{\eta} = \text{argmax}_\eta p(w_j; \eta)$

$\eta := [\theta \ \lambda_j \ \lambda_{k_1} \ \ldots \ \lambda_{k_p} \ \beta_j \ \beta_{k_1} \ \ldots \ \beta_{k_p} \ \sigma_j^2]^T$

Algorithms for multi-stage methods

Two stage method – Empirical Bayes \[^1\]:
- Incorporate Gaussian process modeling and TC kernels in \textit{indirect identification}
- Situation handled of sensor noise only

Model order reduction Steiglitz McBride (MORSM) \[^2\]:
- Step 1: Estimate a high-order ARX model using least squares
- Step 2: Apply SM to the simulated output and filtered input obtained from the estimates
- No non-convex optimization problems involved to get the parametric model

\[^1\] Everitt et al., Automatica 2018.
\[^2\] Galrinho et al., IFAC 2017.
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Network identifiability for a single module

Can **one particular target** module $G_{ji}$ be **distinguished** in network models on the basis of (selected) measured signals $w, r$?
Single module identifiability

Consider a network model set: \( \mathcal{M} = \{(G(\theta), R, H(\theta))\}_{\theta \in \Theta} \)

Based on a subset of measured node signals: \( w_m = Cw \)

Identification algorithms typically can uniquely estimate from \((w_m, r)\):

\[
T_{w_m r} \text{ and } \Phi_{\tilde{v}_m}
\]

with \( w_m = T_{w_m r} r + \tilde{v}_m \)

and \( \Phi_{\tilde{v}_m} \) the power spectral density of \( \tilde{v}_m \)
Single module identifiability

Definition

A module $G_{ji}$ is network identifiable from $(w_m, r)$ in a model set $\mathcal{M}$ at $M_0 = M(\theta_0)$ if for all models $M(\theta_1) \in \mathcal{M}$:

$$
\begin{align*}
T_{w_m r}(q, \theta_1) &= T_{w_m r}(q, \theta_0) \\
\Phi_{\overline{w}_m}(\omega, \theta_1) &= \Phi_{\overline{w}_m}(\omega, \theta_0)
\end{align*}
\implies G_{ji}(\theta_1) = G_{ji}(\theta_0)
$$

It is **globally**\(^{[1]}\) network identifiable if this holds for all $M(\theta_0) \in \mathcal{M}$

It is **generically**\(^{[2]}\) network identifiable if this holds for almost all $M(\theta_0) \in \mathcal{M}$

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\(^{[1]}\) Weerts et al., SYSID2015; Automatica, 2018; CDC 2018

\(^{[2]}\) Bazanella et al., CDC 2017; Hendrickx et al., IEEE-TAC, 2019.; Weerts et al., CDC 2018
Single module identifiability

- **Global** identifiability: dependent on rank conditions
- **Generic** identifiability: path-based conditions on the network graph \([1],[2]\)

Generic rank = number of vertex-disjoint paths

\[ b_{R \rightarrow W} = 3 \]

Single module identifiability

Aspects / situations to be distinguished:

- Partial or full node measurements $w_m = w$
- Partial or full external excitation through $r$: $R = I$
- When discarding the spectrum equality$^1$:

$$\begin{align*}
T_{w_m r}(q, \theta_1) &= T_{w_m r}(q, \theta_0) \\
\Phi_{v_m}(\omega, \theta_1) &= \Phi_{v_m}(\omega, \theta_0)
\end{align*} \implies G_{ji}(\theta_1) = G_{ji}(\theta_0)$$

one only exploits excitation from $r$ rather than from $(r, e)$: cf. indirect/direct method

$^1$ Bazanella et al., CDC 2017; Hendrickx et al., IEEE-TAC, 2019.
Single module identifiability

**Particular result:** full measurement, partial excitation through \( r \) [1]:

For **generic** identifiability of target module:
- Measure all node signals in the network
- Excite a number of ascendants of the in-neighbours of \( w_j \) such that
  \[
  b_{\mathcal{R}\rightarrow \mathcal{N}} = b_{\mathcal{R}\rightarrow \mathcal{N}\setminus\{w_i\}} + 1
  \]

## Single module identifiability

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<th>Excitation conditions</th>
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<td>r</td>
<td>r,e</td>
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<td>Weerts et al. (Autom 2018) - global</td>
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<td>Weerts et al. (CDC, 2018) - global, generic</td>
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<td><strong>Partial excitation - partial measurement</strong></td>
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</table>

Conditions for consistent module estimates (indirect/direct/generalized) act as sufficient conditions for single module identifiability
Extensions - Summary
Extensions

- **Optimal experiment design**, when excitation signal locations have been chosen
  Gevers et al., 2015; Bombois et al., 2018; Morelli et al., 2019;

- **Handling of sensor noise**, leading to errors-in-variables problems
  Dankers et al., 2015;

- **Variance aspects of estimation in structured models**
  Wahlberg et al., 2009; Günes et al., 2014; Everitt et al., 2013, 2017;
Summary

- Path-based conditions for consistent identification
- Degrees of freedom in selection of location for sensing/actuation
- Algorithms that avoid large scale non-convex optimization
- Important aspect: effectively using disturbances for exciting the network related to choice of indirect / direct / generalized method
- A priori known modules can be accounted for
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