EINDHOVEN UNIVERSITY OF TECHNOLOGY

Single module identification in dynamic networks – the current status

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21st IFAC World Congress , 12-17 July 2020, Berlin, Germany

Open invited track:
 “Data-driven modeling and learning in dynamic networks”

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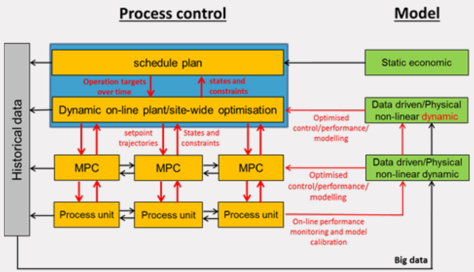
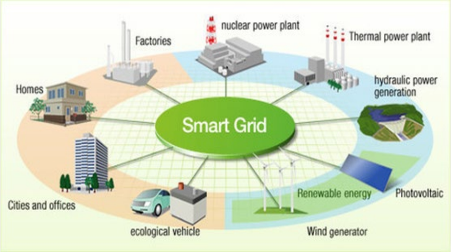
European Research Council

Introduction – dynamic networks

Decentralized process control

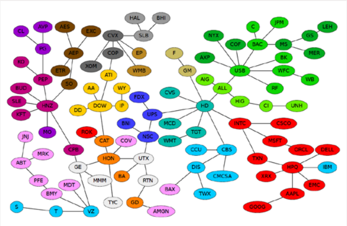


Smart power grid



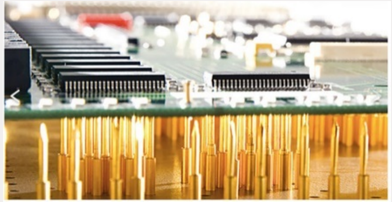
Betterworldsolutions.eu

Stock market



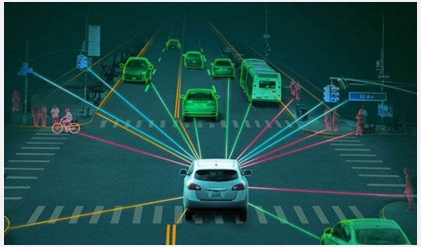
Materassi and Innocenti, 2010

PCB testing



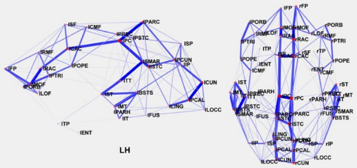
T&M Solutions, Romex BV

Autonomous driving



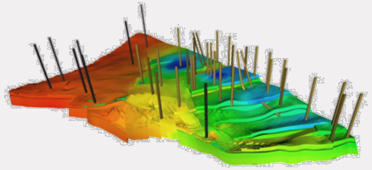
www.envidia.com

Brain network



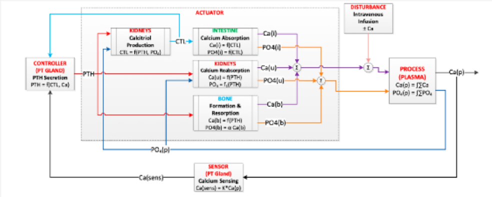
P. Hagmann et al. (2008)

Hydrocarbon reservoirs



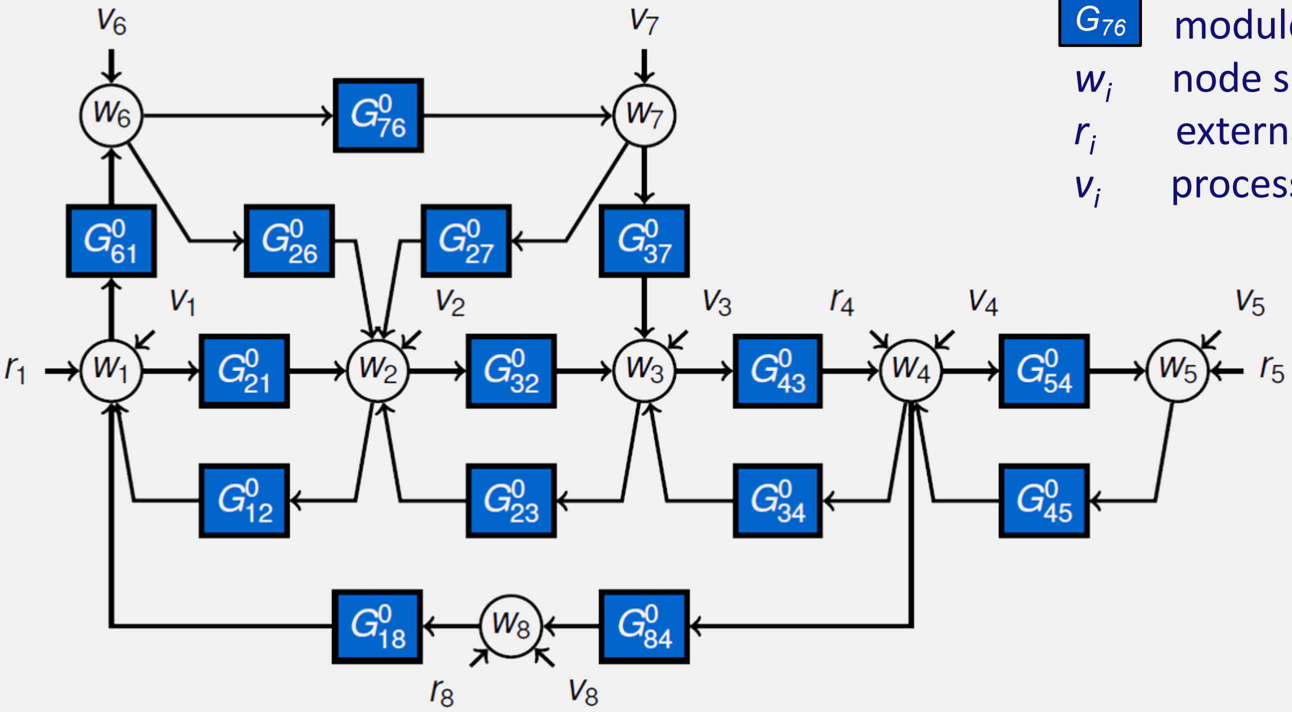
Mansoori (2014)

Physiological models



Christie, Achenie and Ogunnaike (2014)

Dynamic network setup



G_{76} module

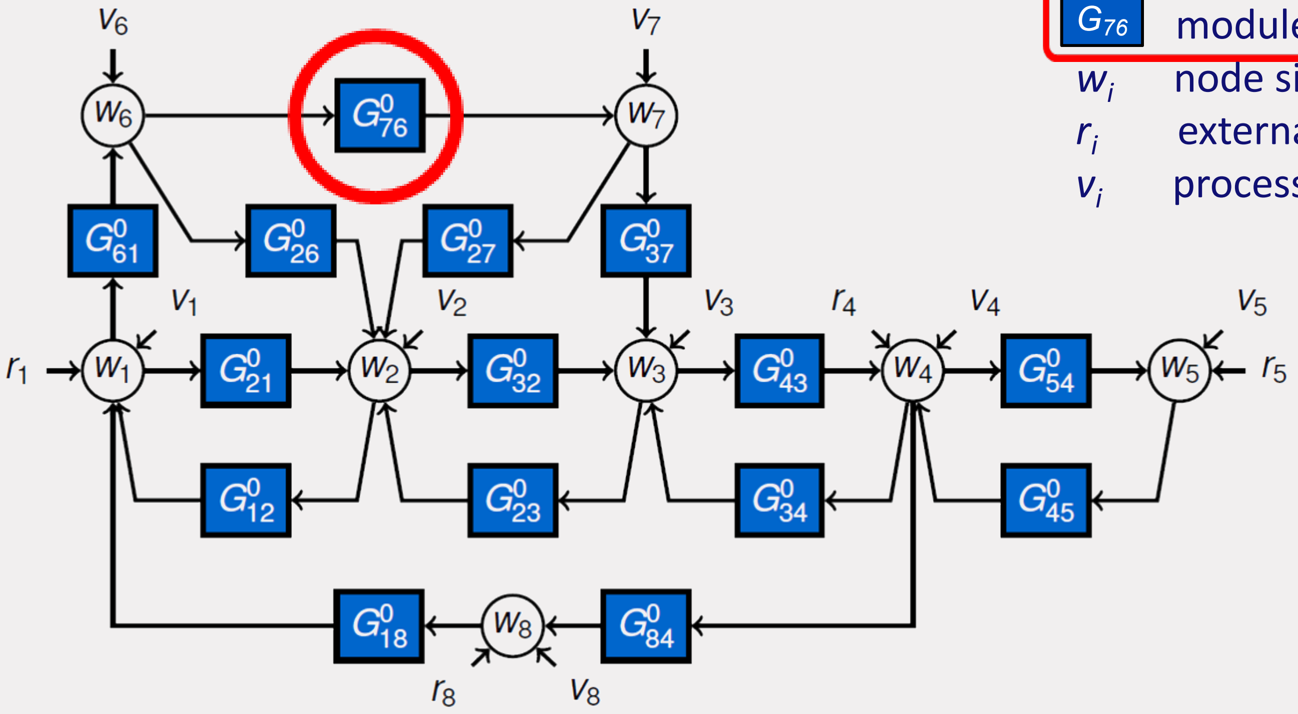
w_i node signal

r_i external excitation

v_i process noise

TU/e

Dynamic network setup



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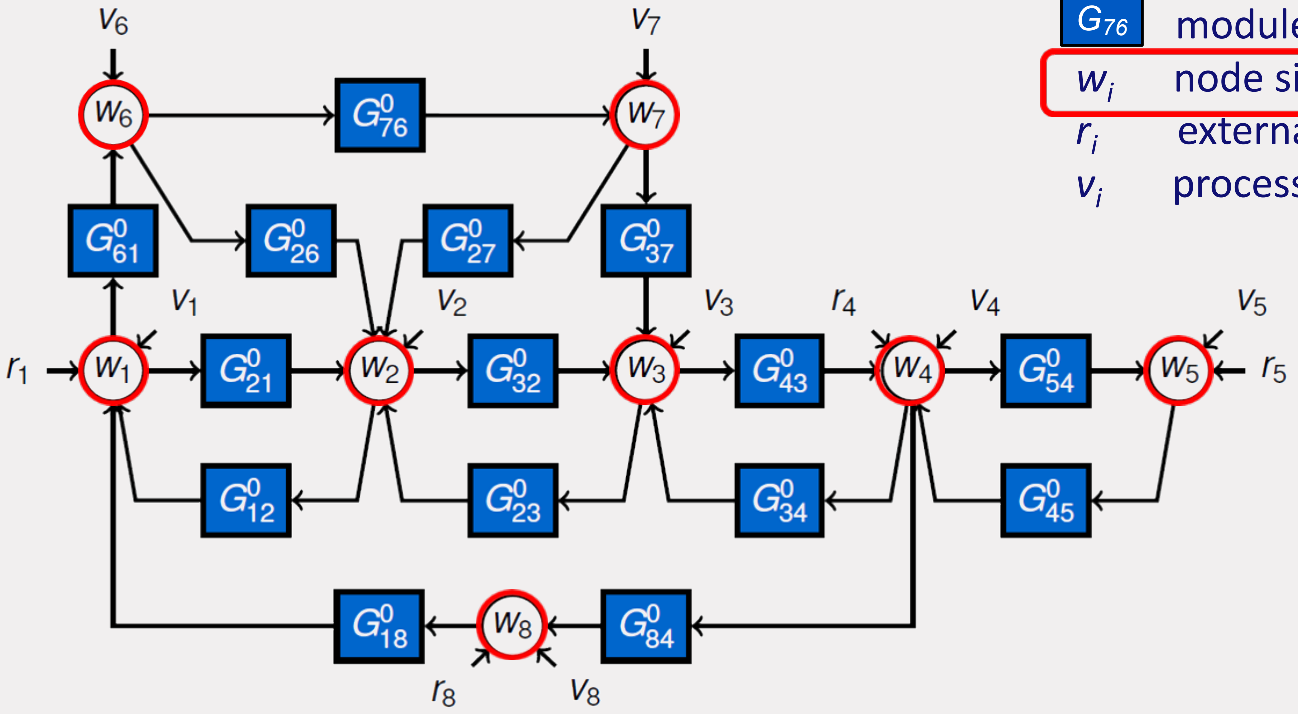

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TU/e

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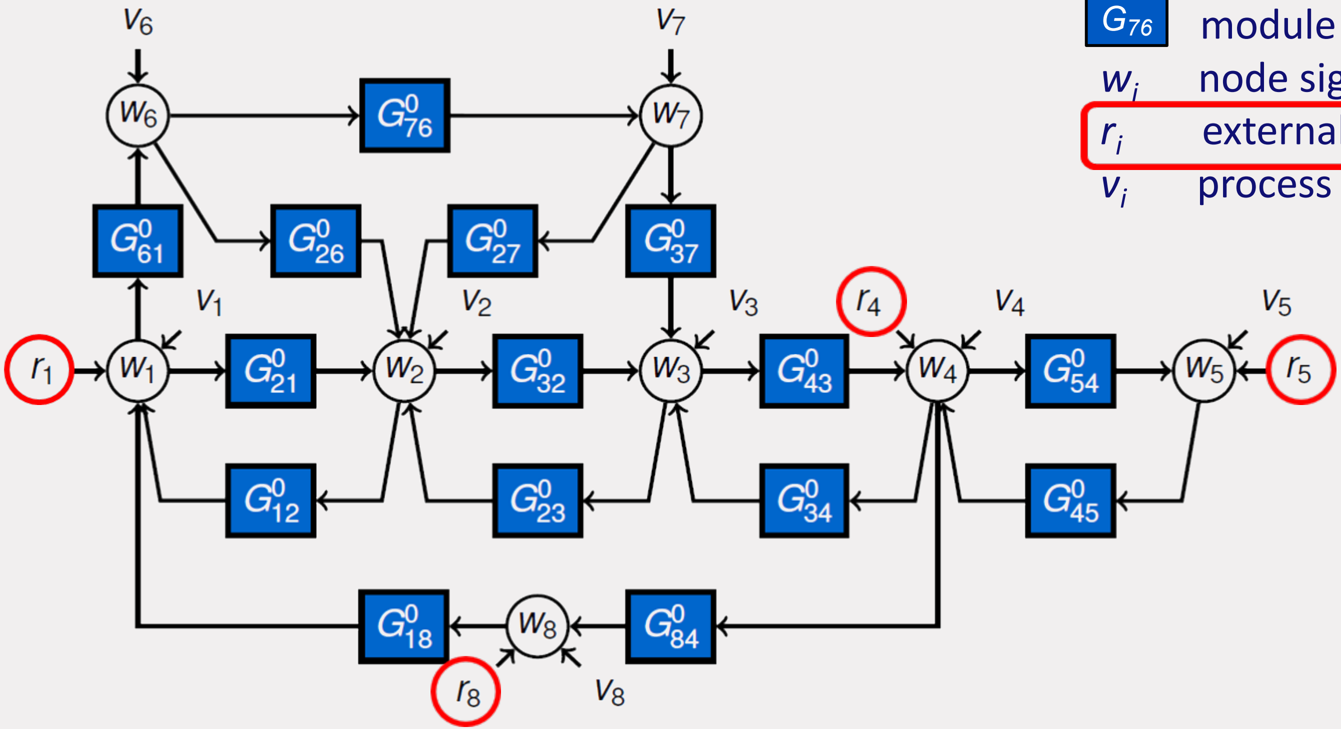

G_{76} module

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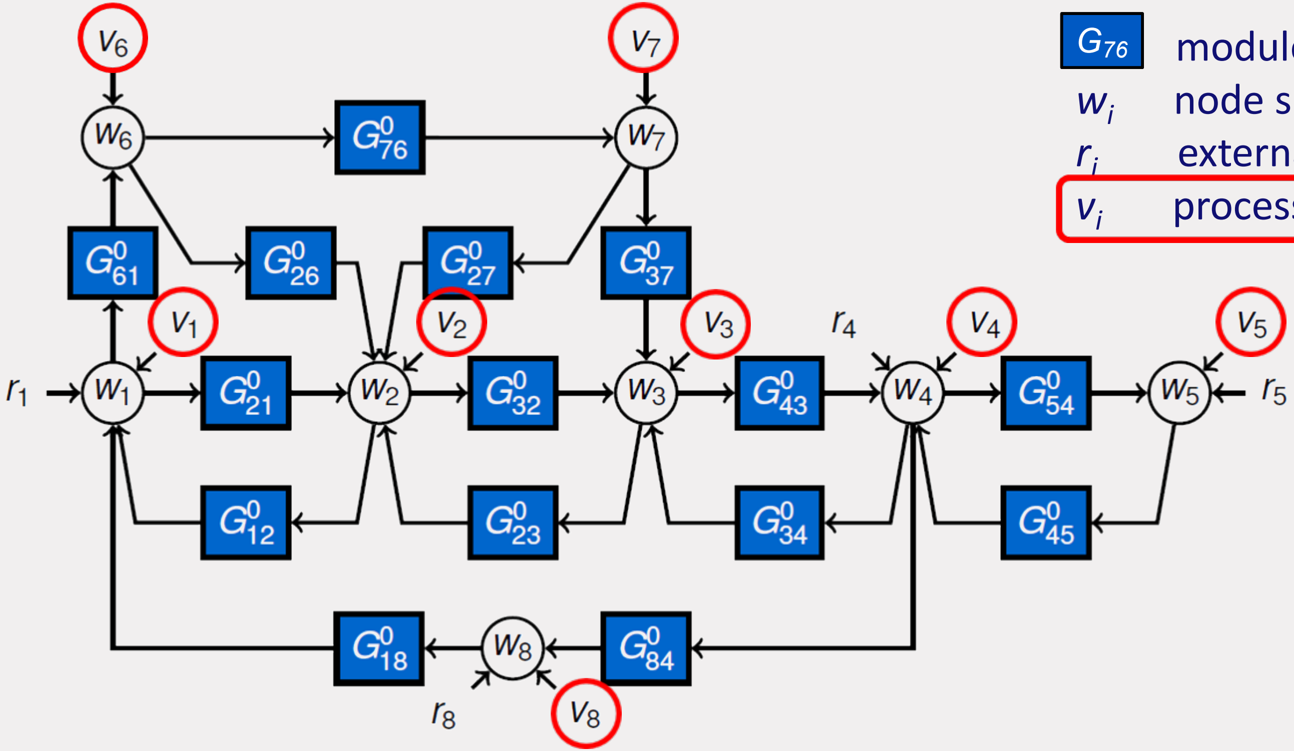
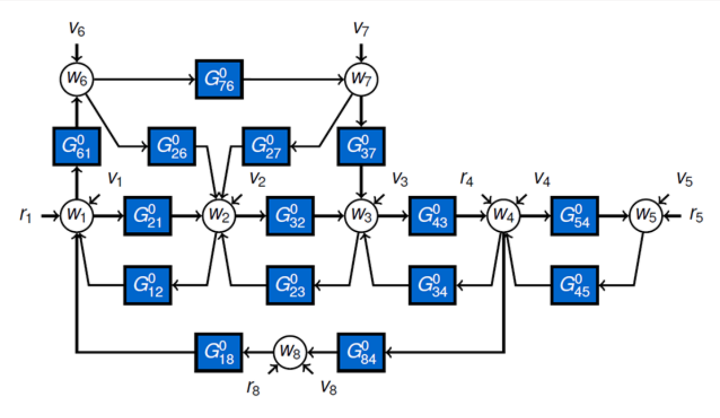
Dynamic network setup



G_{76}	module
w_i	node signal
r_i	external excitation
v_i	process noise

Dynamic network setup



Assumptions:

- Total of L nodes
- Network is well-posed and stable
- Modules are dynamic LTI, may be unstable
- Disturbances are stationary stochastic and can be correlated

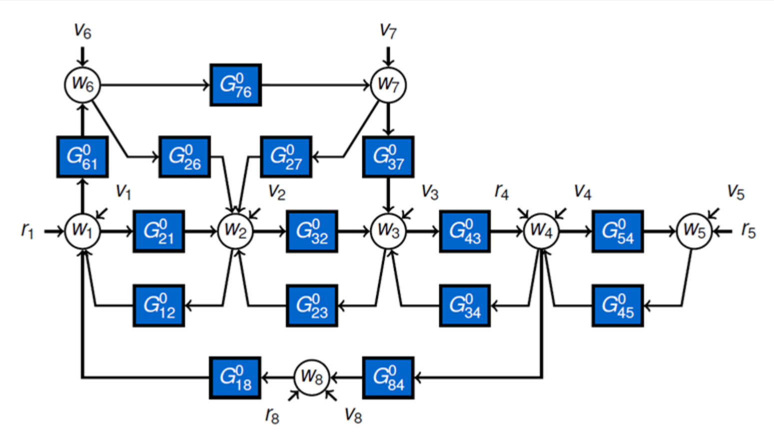
$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \ddots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix}}_{G^0(q)} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + R^0 \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}$$

q is forward time shift

$$w(t) = G^0(q)w(t) + R^0(q)r(t) + v(t) \qquad v(t) = H^0(q)e(t)$$

[1] J. Gonçalves and S. Warnick, IEEE TAC, 2008.
[2] PVdH et al., Automatica, 2013.

Dynamic network setup



Measured time series:

$$\{w_i(t)\}_{i=1,\cdots L}; \ \{r_j(t)\}_{j=1,\cdots K}$$

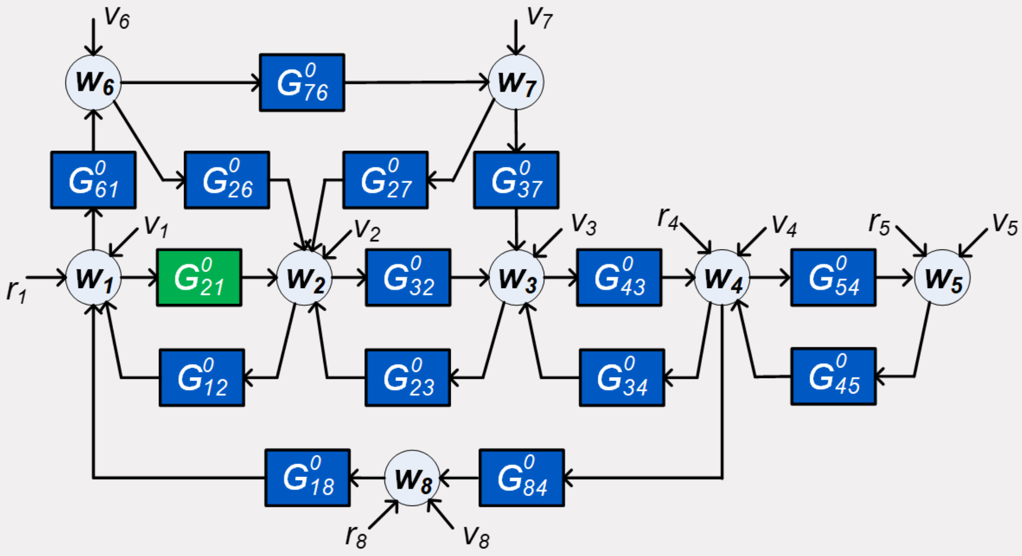
Many data-driven modeling questions can be formulated

- **Identification of a local module (known topology)**
- Identification of the full network
- Topology estimation
- Identifiability
- Sensor and excitation allocation
- Fault detection
- User prior knowledge of modules
- Distributed identification
- Scalable algorithms

Contents

- Introduction and network model
- Single module identification: what’s the problem?
- Indirect methods
- Direct methods
- Algorithmic aspects
- Single module identifiability
- Conclusions

Single module identification



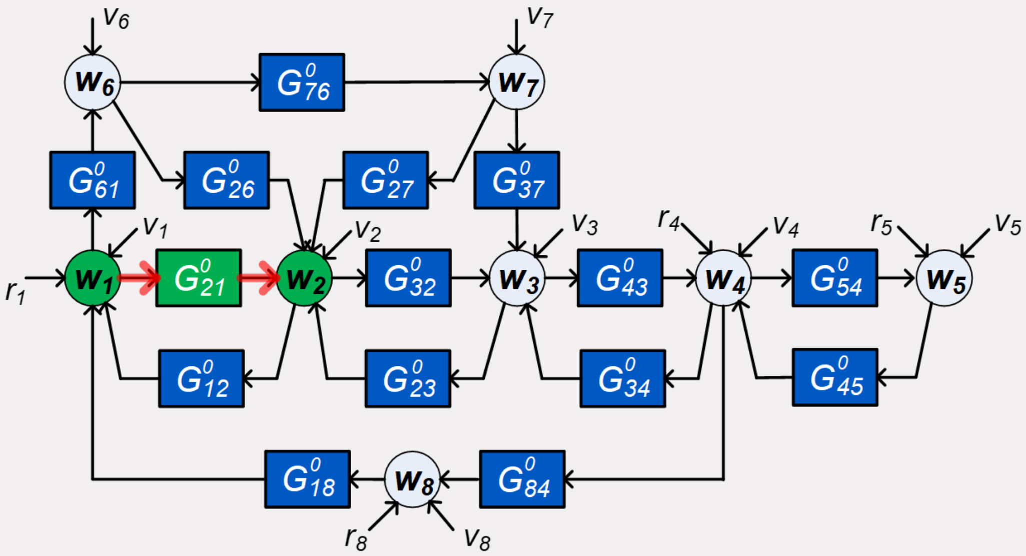
The problem:

For a network with known topology:

Identify G^0_{21} on the basis of selected measured signals (w, r)

Preference for "local" measurements and limited excitation

Single module identification



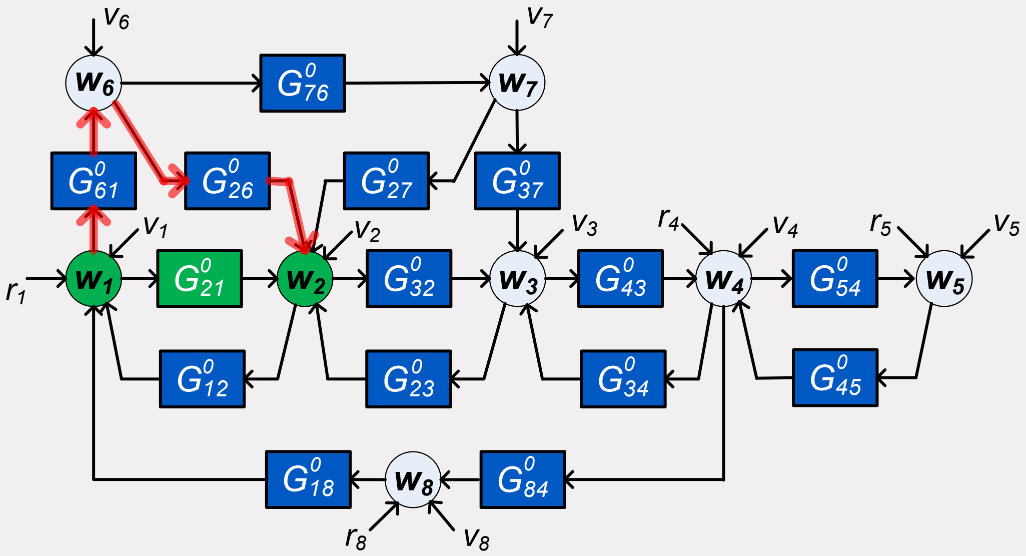
Naïve approaches:

- identify based on w_2 and w_1 ; or
- identify based on $T_{w_2 r_1} T^{-1}_{w_1 r_1}$

do not work,

e.g. because of parallel paths

Single module identification



Naïve approaches:

- identify based on w_2 and w_1 ; or
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do not work,

e.g. because of parallel paths

Single module identification

Approaches to the problem:

1. **Prediction error methods**

   VdH et al. (2013); Dankers et al. (2015, 2016); Galrinho et al. (2017); Everitt et al. (2018); Gevers et al. (2018); Bazanella et al. (2017, 2019), Hendrickx et al. (2019), Ramaswamy et al. (2018, 2019, 2020);

   generalizations of closed-loop methods, requiring choice of predictor model

2. **Alternatives**

   - Non-parametric methods, based on Wiener filters and d-separation

     Materassi & Salapaka, (2015,2020)

   - Subspace methods

     Yu and Verhaegen, TAC (2018)

Single module identification

Prediction error methods:

Choice of predictor model, leading to prediction errors:

Direct method: $\varepsilon(t,\theta) = \bar{H}(q,\theta)^{-1}[w_{\mathcal{Y}}(t) - \bar{G}(q,\theta)w_{\mathcal{D}}(t)]$

direct estimation of target module

Indirect method: $\varepsilon(t,\theta) = \bar{H}(q,\theta)^{-1}[w_{\mathcal{Y}}(t) - \bar{T}(q,\theta)r_{\mathcal{D}}(t)]$

indirect estimation through post-processing

Generalized method: $\varepsilon(t,\theta) = \bar{H}(q,\theta)^{-1}[w_{\mathcal{Y}}(t) - \bar{G}(q,\theta)w_{\mathcal{D}_w}(t) - \bar{T}(q,\theta)r_{\mathcal{D}_r}(t)]$

Single module identification

Prediction error methods:

Main differences:

Direct method: $\varepsilon(t,\theta) = \bar{H}(q,\theta)^{-1}[w_{\mathcal{Y}}(t) - \bar{G}(q,\theta)w_{\mathcal{D}}(t)]$

Predictor inputs $w_{\mathcal{D}}(t)$ receive excitation from both r and e signals

Indirect method: $\varepsilon(t,\theta) = \bar{H}(q,\theta)^{-1}[w_{\mathcal{Y}}(t) - \bar{T}(q,\theta)r_{\mathcal{D}}(t)]$

Predictor inputs $r_{\mathcal{D}}(t)$ receive excitation from r signals only

Overall: indirect methods have stronger requirements on the presence of measurable external excitation signals r → more expensive experiments

TU/e

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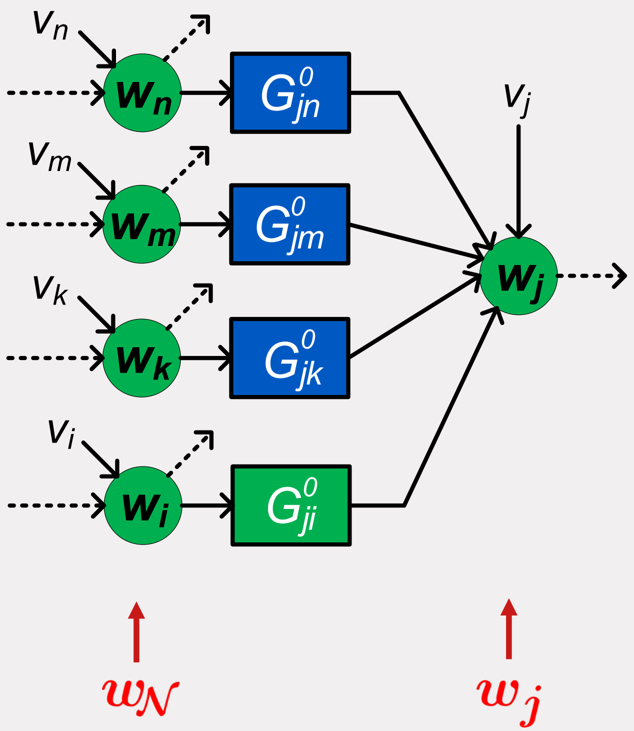
Single module identification

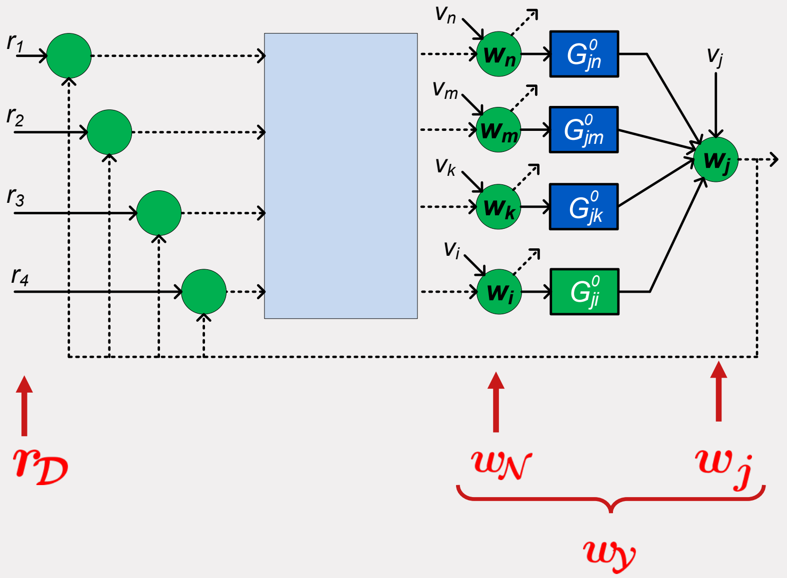


Multi-input single-output identification problem

to be addressed by a closed-loop identification method

Indirect methods

How to choose predictor inputs and outputs?



MISO identification problem

- Select output w_j and all its in-neighbors $w_{\mathcal{N}}$ as predictor output; $r_{\mathcal{D}}$ as predictor input
- Estimate $\bar{T}_{\mathcal{N}r}$ and $\bar{T}_{jr}$ consistently, and determine:

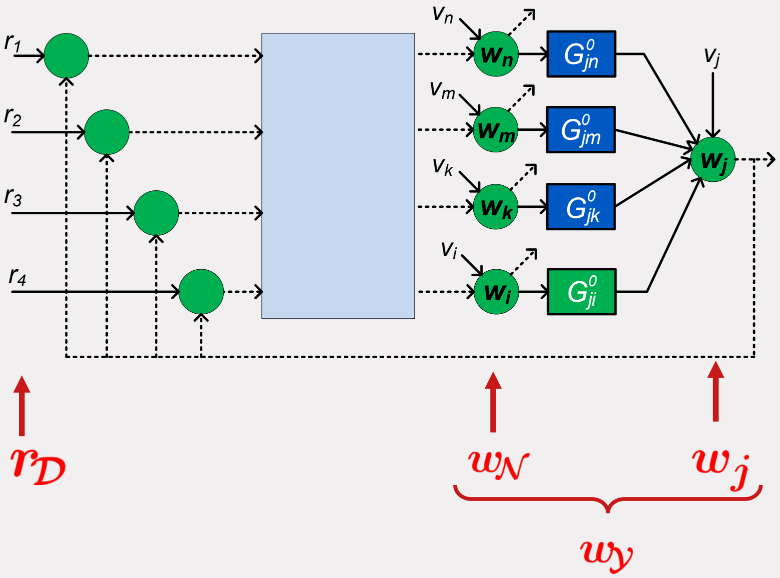
$$\hat{G}_{j\mathcal{N}} = \hat{T}_{jr}\hat{T}_{\mathcal{N}r}^{-1} \qquad [1]$$

- or through IV or two-stage method[2]
- freedom in location of r-signals (e.g. directly on $w_{\mathcal{N}}$)
- dual (outneighbour) setup is also possible[1]
- we do not necessarily need all in-neighbors to be included in $w_{\mathcal{N}}$

[1] Gevers et al., SYSID 2018; Hendrickx et al, TAC 2019; Bazanella et al., CDC 2019
[2] VdHof et al., Automatica 2013; Dankers et al., Automatica 2015

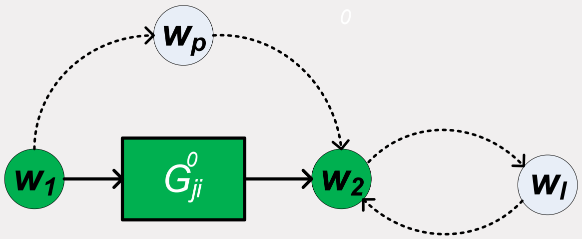
Indirect methods

How to choose predictor inputs and outputs?



Selection of signals in $w_{\mathcal{Y}}$:

- Parallel path and loop condition

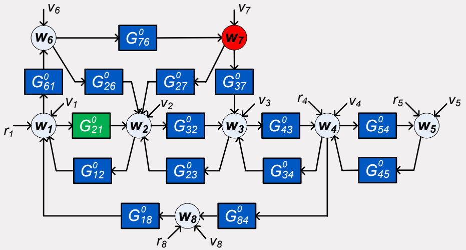


All parallel paths, and loops around the output, should pass through a signal in $w_{\mathcal{Y}}$

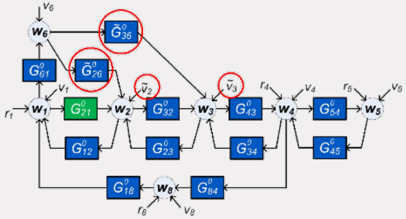
[1] Dankers et al., IEEE-TAC, 2016

Indirect methods

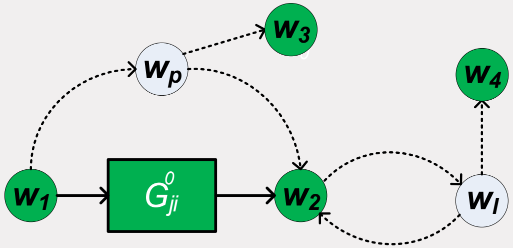
- Parallel path and loop condition results from theory of **immersion**[1]**:** removing node signals, while retaining the behaviour of the remaining nodes



Immersing w_7



With network **abstractions**[2] this can further be generalized:



Measuring descendants of the requested nodes instead

[1] Dankers et al., IEEE-TAC, 2016; F. Dörfler and F. Bullo, 2013

[2] Linder and Enqvist, 2017; Weerts, Linder et al., Automatica, July 2020

Indirect methods

- Relatively simple methods for **consistent estimation** of target module

- High requirements on presence of excitation signals $\boldsymbol{r}$

  leading to "expensive" experiments

  No use of excitation through disturbance signals

As alternative: direct method

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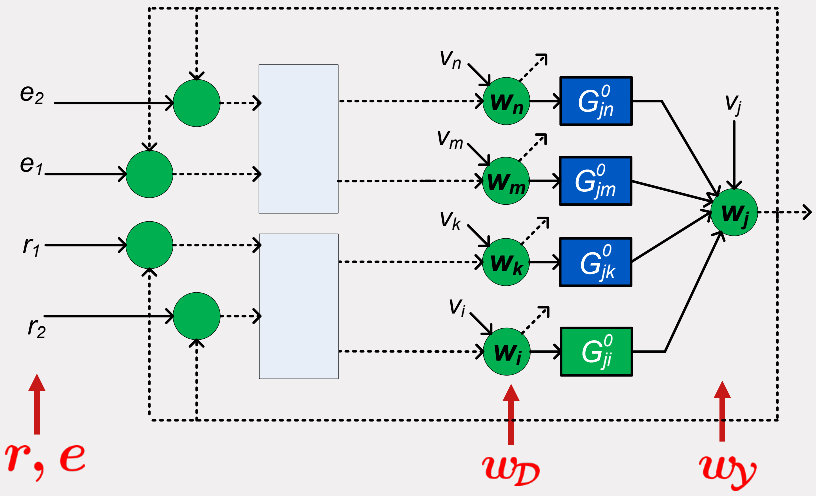
Direct method



$$\varepsilon(t,\theta) = \bar{H}(q,\theta)^{-1}[w_{\mathcal{Y}}(t) - \bar{G}(q,\theta)w_{\mathcal{D}}(t)]$$

- Estimate transfer $w_{\mathcal{D}} \rightarrow w_{\mathcal{Y}}$ and model the disturbance process on the output.
- consistent estimate and ML properties
- provided there is enough excitation, through external signals r and e
- input signal set $w_{\mathcal{D}}$ can be further reduced[1]

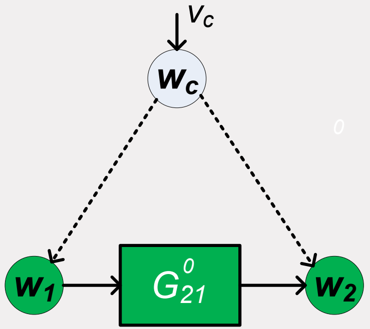
Additional problem:

If:
- v signals are correlated, i.e. $\Phi_v(\omega)$ non-diagonal, or
- some in-neighbors of $w_{\mathcal{Y}}$ are not included in $w_{\mathcal{D}}$

Then confounding variables can occur, destroying the consistency results

[1] Dankers et al., IEEE-TAC, 2016; Dankers et al., IFAC 2017

Direct method



Confounding variable [1][2]**:**

Unmeasured signal that has (unmeasured paths) to both the input and output of an estimation problem.

Can be addressed in two ways[3]:

- by adding an additional node signal to $w_{\mathcal{D}}$, and blocking an unmeasured path;

  OR

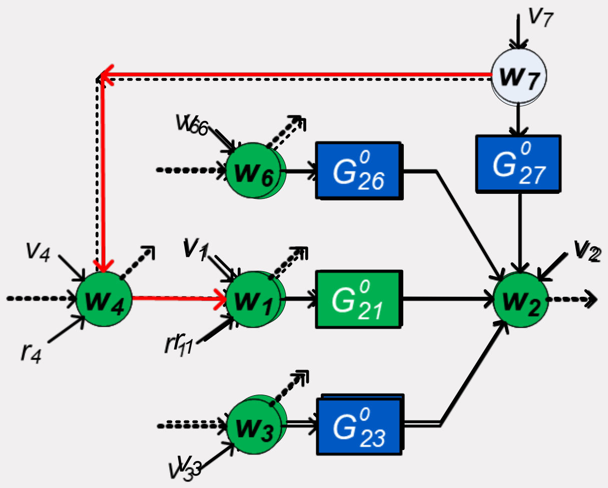
- by adding the affected signal in $w_{\mathcal{D}}$ to $w_{\mathcal{Y}}$ and model the correlated disturbances

Resulting predictor model can become a **MIMO model**

[1] J. Pearl, *Stat. Surveys, 3,* 96-146, 2009
[2] A.G. Dankers et al., *Proc. IFAC World Congress*, 2017.
[3] PVdH et al, CDC 2019; Ramaswamy et al., 2020

Direct method

Example of confounding variable handling:



Non-measurable w_7 is a confounding variable

Two possible solutions:

1. Include w_4 ➡ add predictor input

$$w_{\mathcal{D}} = \{w_1, w_3, w_4, w_6\} \quad w_{\mathcal{Y}} = \{w_2\}$$

2. Predict w_1 too ➡ add predictor output

$$w_{\mathcal{D}} = \{w_1, w_3, w_6\} \quad w_{\mathcal{Y}} = \{w_1, w_2\}$$

Relation with d-separation in graphs (Materassi & Salapaka)[1]

[1] D. Materassi and M.V. Salapaka, CDC 2015, TAC 2020.

Direct method - Algorithm for signal selection

For estimating target module G_{ji}:

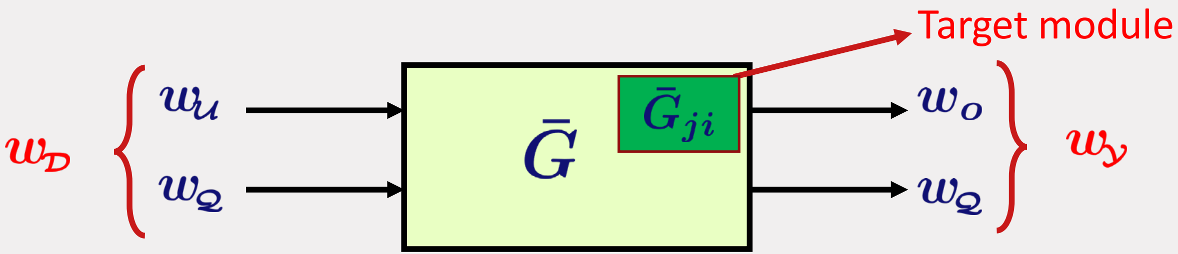
1. Select $w_{\mathcal{D}} = w_i$ and $w_{\mathcal{Y}} = w_j$
2. Add node signals to $w_{\mathcal{D}}$ to satisfy the parallel path and loop condition
3. Extend $w_{\mathcal{D}}$ and / or $w_{\mathcal{Y}}$ so as to avoid confounding variables

Algorithm always reaches a convergence point where conditions are satisfied.

The choice options lead to different end-results for signals to be included
→ different predictor models

Direct method

General setup:

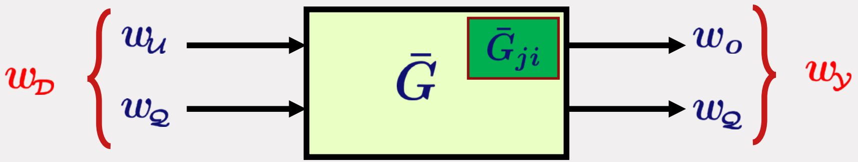


Different predictor models:

- Full input case : include all in-neighbors of $w_{\mathcal{Y}}$
- Minimum node signals case : maximize number of outputs
- User selection case : dedicated choice based on measurable nodes

Consistency result

Conditions for consistent (and ML) estimation of G_{ji} [1]:



- System in the model set,
- Parallel path and loop condition satisfied
- Confounding variables handled appropriately
- Persistence of excitation, i.e. $\Phi_{\kappa}(\omega) > 0$ at a sufficient number of frequencies, with

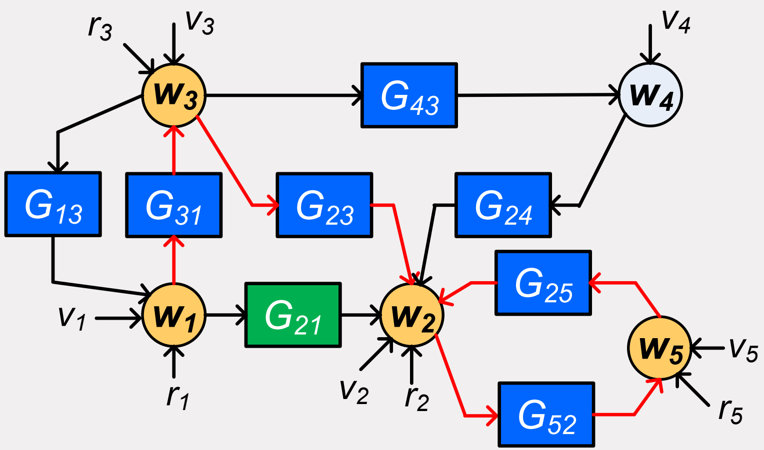
  $$\kappa = \begin{bmatrix} w_{\mathcal{D}} \\ \xi_{\mathcal{Q}} \\ w_{\mathcal{O}} \end{bmatrix}$$ and $\xi_{\mathcal{Q}}$ the innovation process of $w_{\mathcal{Q}}$

  (can also be phrased as path-based condition[2])

- Requirements on signals r increase with increasing number of outputs

[1] K.R. Ramaswamy et al., ArXiv 2019, IEEE-TAC, provis accepted.
[2] VdH et al., CDC-2020 submitted

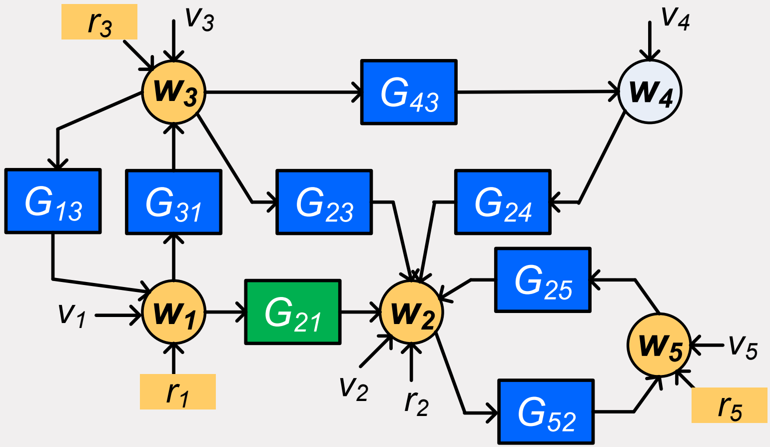
Example - direct method & indirect method



$$\underbrace{\overset{w_{\mathcal{D}}}{\{w_1, w_3, w_5\}} \rightarrow \overset{w_{\mathcal{Y}}}{\{w_2\}}}_{Direct\ \ method\ [1]}$$

[1] A. Dankers et al., TAC 2016.

Example - direct method & indirect method



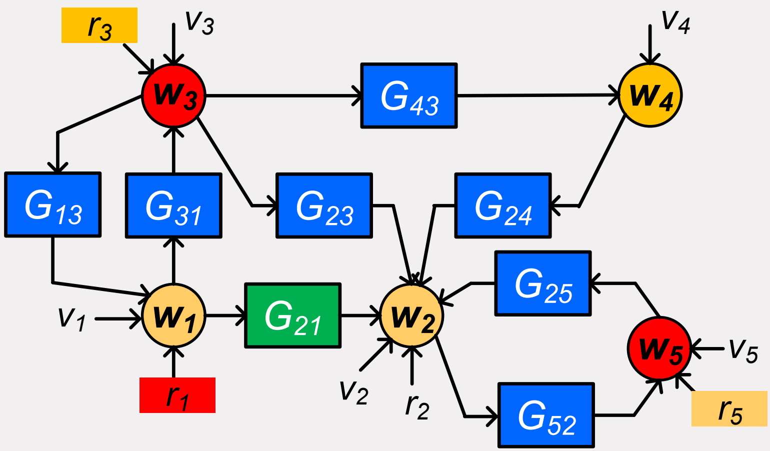
$$\underbrace{\overset{\color{red}{w_{\mathcal{D}}}}{\{w_1, w_3, w_5\}} \rightarrow \overset{\color{red}{w_{\mathcal{Y}}}}{\{w_2\}}}_{\textit{Direct method}\ [1]}$$

$$\underbrace{\overset{\color{red}{r_{\mathcal{D}}}}{\{r_1, r_3, r_5\}} \rightarrow \overset{\color{red}{w_{\mathcal{Y}}}}{\{w_1, w_2, w_3, w_5\}}}_{\textit{Indirect method}\ [2]}$$

[1] A. Dankers et al., TAC 2016.

[2] M. Gevers, et al., SYSID 2018.

Example - direct method & indirect method



$$\underbrace{\{w_1, w_3, w_5\} \rightarrow \{w_2\}}_{Direct\ \ method}$$

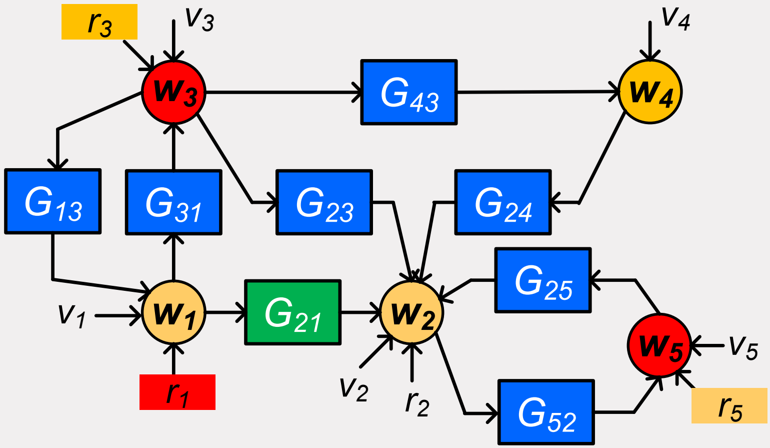
$$\underbrace{\{r_1, r_3, r_5\} \rightarrow \{w_1, w_2, w_3, w_5\}}_{Indirect\ \ method}$$

- What can we do if parallel path/loop conditions cannot be satisfied?
- What can we do if certain nodes cannot be excited?

We combine the ideas of direct and indirect methods to increase flexibility

TU/e

Example - direct method & indirect method



$$\underbrace{\{w_1, w_3, w_5\} \rightarrow \{w_2\}}_{Direct\ \ method}$$

$$\underbrace{\{r_1, r_3, r_5\} \rightarrow \{w_1, w_2, w_3, w_5\}}_{Indirect\ \ method}$$

$$\underbrace{\{w_1, w_4, r_2, r_3\} \rightarrow \{w_2, w_4\}}_{Generalized\ \ method}$$ [1]

- Include both internal nodes and external excitation as predictor inputs
- Instead of measuring a parallel path we excite it and measure a descendant
- Generalized method increases flexibility in selecting sensors/actuators

[1] K.R. Ramaswamy et al., CDC 2019

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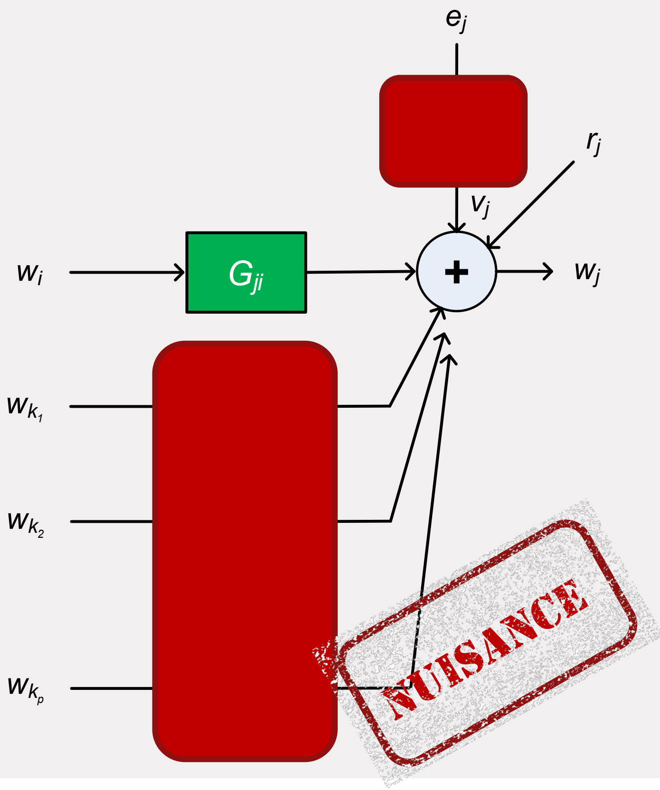
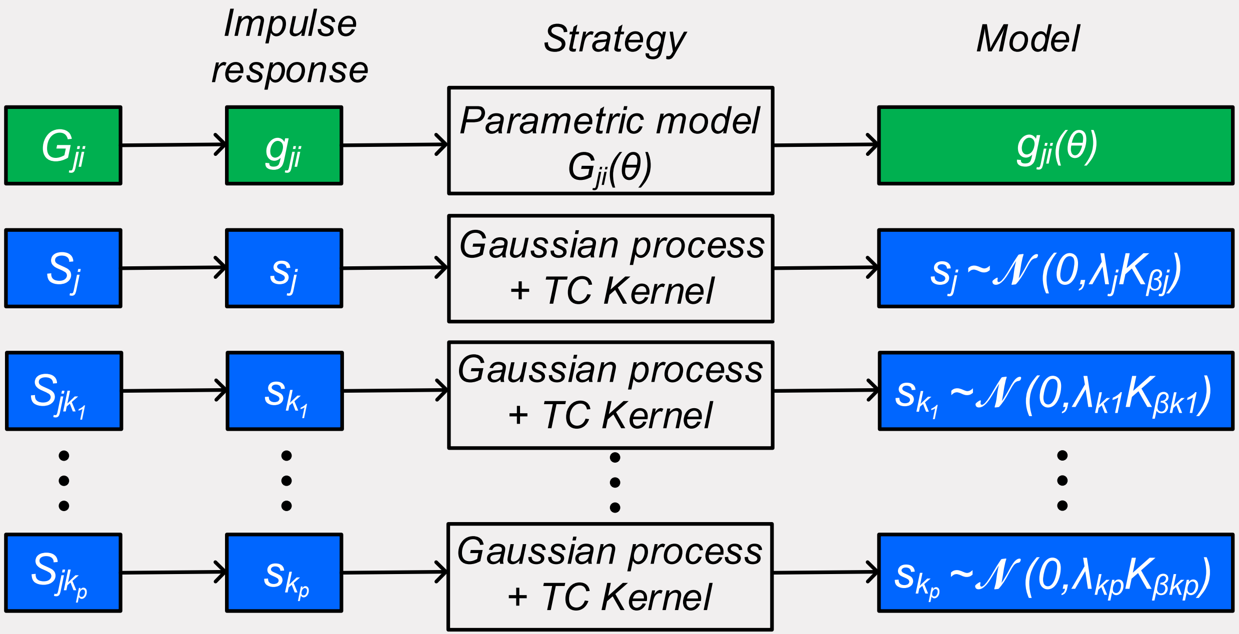
Machine learning in local module identification

- MISO/MIMO identification with all modules parameterized
- Brings in some major computational complexity

- We need only the target module. No NUISANCE!



Machine learning in local module identification



- smaller no. of parameters
- simpler model order selection
- scalable
- simpler optimization problems to estimate parameters

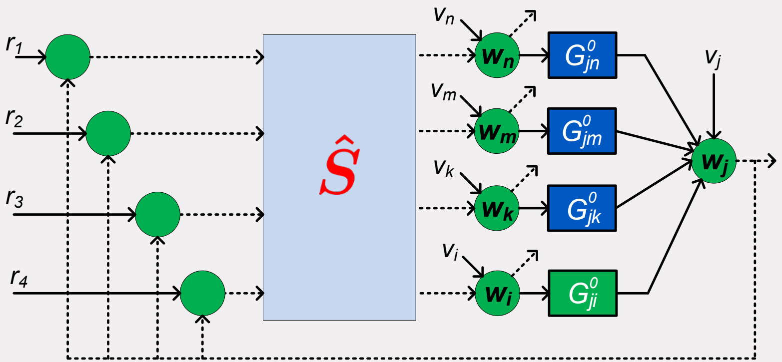
Maximize marginal likelihood of output data: $\hat{\eta} = \underset{\eta}{\operatorname{argmax}}\, p(w_j; \eta)$

$$\eta := \begin{bmatrix} \theta & \lambda_j & \lambda_{k_1} & \cdots & \lambda_{k_p} & \beta_j & \beta_{k_1} & \cdots & \beta_{k_p} & \sigma_j^2 \end{bmatrix}^\top$$

Algorithms for multi-stage methods

Two stage method – Empirical Bayes[1]:

- Incorporate Gaussian process modeling and TC kernels in **indirect identification**
- Situation handled of sensor noise only



Model order reduction Steiglitz McBride (MORSM)[2]:

- Step 1: Estimate a high-order ARX model using least squares
- Step 2: Apply SM to the simulated output and filtered input obtained from the estimates
- No non-convex optimization problems involved to get the parametric model

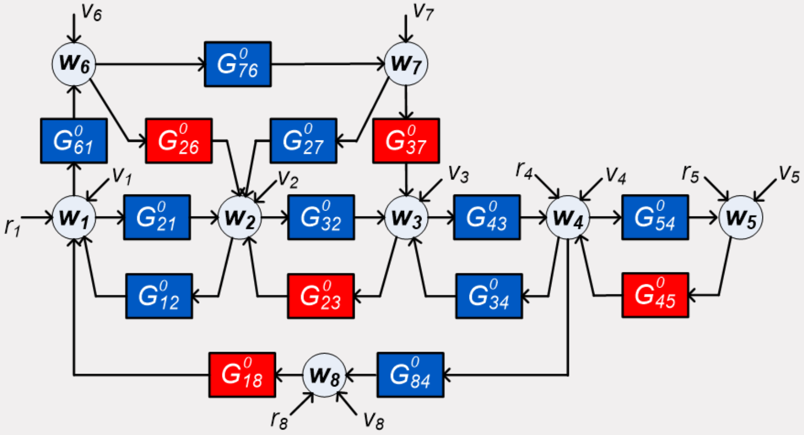
[1] Everitt et al., Automatica 2018.

[2] Galrinho et al., IFAC 2017.

Contents

- Introduction and network model
- Single module identification: what’s the problem?
- Indirect methods
- Direct methods
- Algorithmic aspects
- Single module identifiability
- Conclusions

Network identifiability for a single module



blue = unknown
red = known

Can **one particular target** module G_{ji} be **distinguished** in network models on the basis of (selected) measured signals w, r ?

Single module identifiability

Consider a network model set: $\mathcal{M} = \{(G(\theta), R, H(\theta))\}_{\theta\in\Theta}$

Based on a subset of measured node signals: $w_m = Cw$

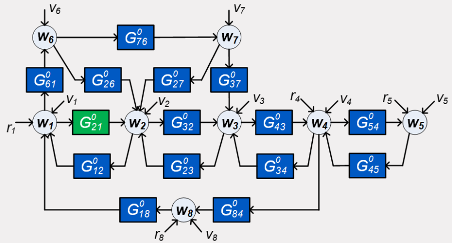
Identification algorithms typically can uniquely estimate from (w_m, r) :

$$\boxed{T_{w_m r} \text{ and } \Phi_{\bar{v}_m}}$$

with $w_m = T_{w_m r} r + \bar{v}_m$

and $\Phi_{\bar{v}_m}$ the power spectral density of $\bar{v}_m$

Single module identifiability



Definition

A module G_{ji} is network identifiable from (w_m, r) in a model set $\mathcal{M}$ at $M_0 = M(\theta_0)$ if for all models $M(\theta_1) \in \mathcal{M}$:

$$\left.\begin{array}{c} T_{w_m r}(q,\theta_1) = T_{w_m r}(q,\theta_0) \\ \Phi_{\bar{v}_m}(\omega,\theta_1) = \Phi_{\bar{v}_m}(\omega,\theta_0) \end{array}\right\} \Longrightarrow G_{ji}(\theta_1) = G_{ji}(\theta_0)$$

It is **globally**[1] network identifiable if this holds for **all** $M(\theta_0) \in \mathcal{M}$

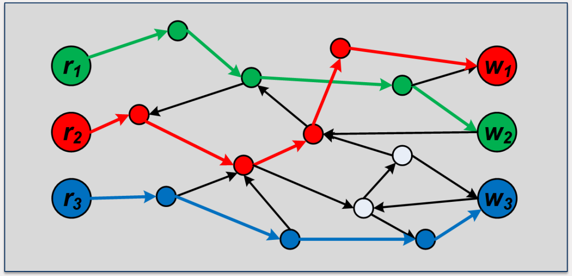
It is **generically**[2] network identifiable if this holds for **almost all** $M(\theta_0) \in \mathcal{M}$

[1] Weerts et al., SYSID2015; Automatica, 2018; CDC 2018
[2] Bazanella et al., CDC 2017; Hendrickx et al., IEEE-TAC, 2019.; Weerts et al., CDC 2018

Single module identifiability

- **Global** identifiability: dependent on **rank** conditions
- **Generic** identifiability: path-based conditions on the network graph [1],[2]

Generic rank = number of vertex-disjoint paths



$$b_{\mathcal{R}\to\mathcal{W}} = 3$$

[1] Van der Woude, 1991
[2] Hendrickx, Gevers & Bazanella, CDC 2017, TAC 2019

Single module identifiability

Aspects / situations to be distinguished:

- Partial or full node measurements $w_m = w$
- Partial or full external excitation through r: $R = I$
- When discarding the spectrum equality[1]:

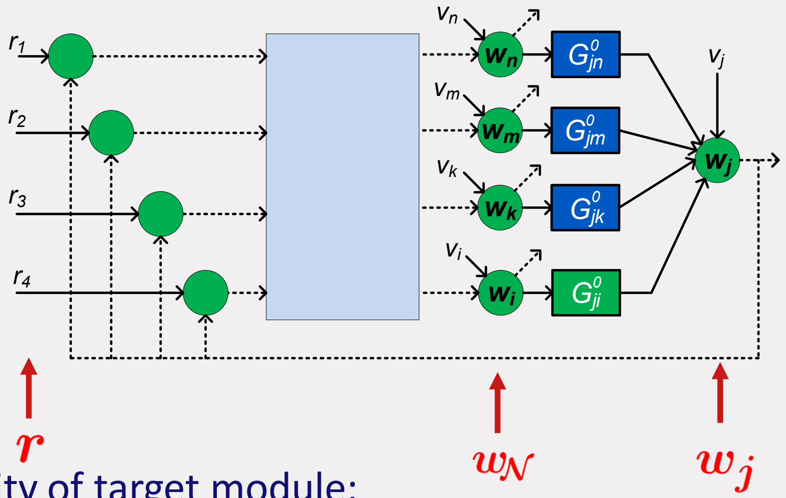
$$\left.\begin{array}{c} T_{w_m r}(q,\theta_1) = T_{w_m r}(q,\theta_0) \\ \cancel{\Phi_{v_m}(\omega,\theta_1) = \Phi_{v_m}(\omega,\theta_0)} \end{array}\right\} \Longrightarrow G_{ji}(\theta_1) = G_{ji}(\theta_0)$$

one only exploits excitation from r rather than from (r,e): cf. indirect/direct method

[1] Bazanella et al., CDC 2017; Hendrickx et al., IEEE-TAC, 2019.

Single module identifiability

Particular result: full measurement, partial excitation through $\boldsymbol{r}$ [1]:



For **generic** identifiability of target module:

- Measure all node signals in the network
- Excite a number of ascendants of the in-neighbours of w_j such that

$$b_{\mathcal{R}\to\mathcal{N}} = b_{\mathcal{R}\to\mathcal{N}\setminus\{w_i\}} + 1$$

[1] Hendrickx, Gevers & Bazanella, CDC 2017, SYSID 2018, TAC 2019

Single module identifiability

	Excitation conditions		
	r	r,e	e
Measurement / excitation setup			
Full measurement - partial excitation	Hendrickx et al. (TAC, 2019) - generic	Weerts et al. (Autom 2018) - global Weerts et al. (CDC, 2018) - global, generic Shi et al. (IFAC, 2020) - generic	--
Full excitation - partial measurement	Bazanella et al. (CDC, 2017) - generic Hendrickx et al. (TAC, 2019) - generic van Waarde et al., (POL, 2018) - global		Materassi & Salapaka (CDC, 2015)
Partial excitation - partial measurement	Bazanella et al. (CDC, 2019) - generic	Analysis through identification methods: VdHof et al. (Autom 2013) - global Ramaswamy et al. (TAC prov accep 2020) - global Ramaswamy et al. (CDC 2019) - global	--

Conditions for consistent module estimates (indirect/direct/generalized) act as sufficient conditions for single module identifiability

Extensions - Summary

Extensions

- Optimal experiment design, when excitation signal locations have been chosen
  Gevers et al., 2015; Bombois et al., 2018; Morelli et al., 2019;

- Handling of sensor noise, leading to errors-in-variables problems
  Dankers et al., 2015;

- Variance aspects of estimation in structured models
  Wahlberg et al., 2009; Günes et al., 2014; Everitt et al., 2013, 2017;

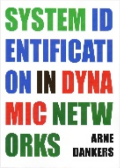
Summary

- Path-based conditions for consistent identification
- Degrees of freedom in selection of location for sensing/actuation
- Algorithms that avoid large scale non-convex optimization
- Important aspect: effectively using disturbances for exciting the network

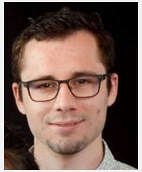
  related to choice of indirect / direct / generalized method
- A priori known modules can be accounted for

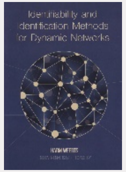
Acknowledgements

Research team:



Arne Dankers





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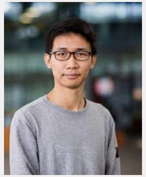

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European Research Council

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Single module identification in dynamic networks – the current status


Paul Van den Hof, Karthik Ramaswamy

21st IFAC World Congress , 12-17 July 2020, Berlin, Germany

Open invited track:
 "Data-driven modeling and learning in dynamic networks"

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