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A regularized kernel-based method for learning a module in a dynamic network with correlated noise

Venkatakrishnan Rajagopal, Karthik Ramaswamy, Paul Van den Hof

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www.sysdynet.eu
k.r.ramaswamy@tue.nl

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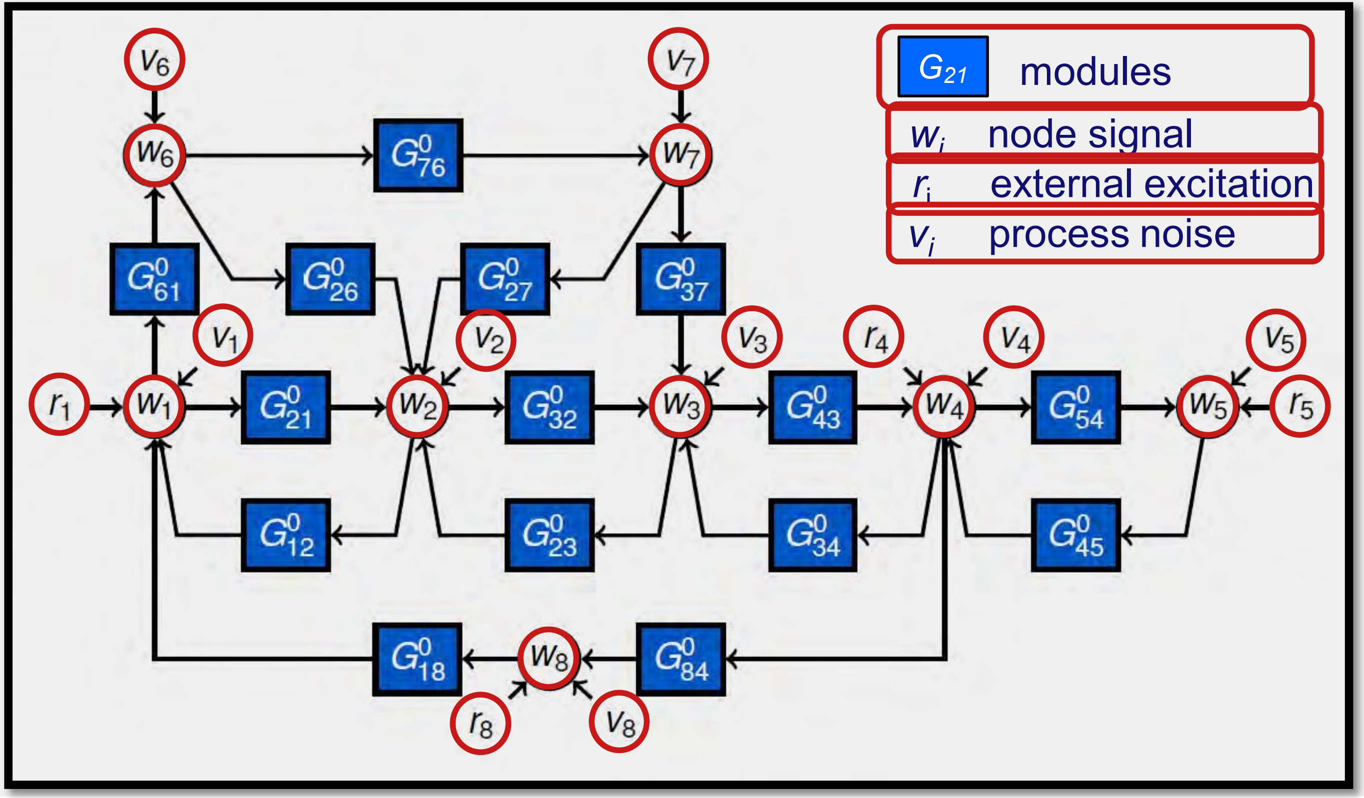
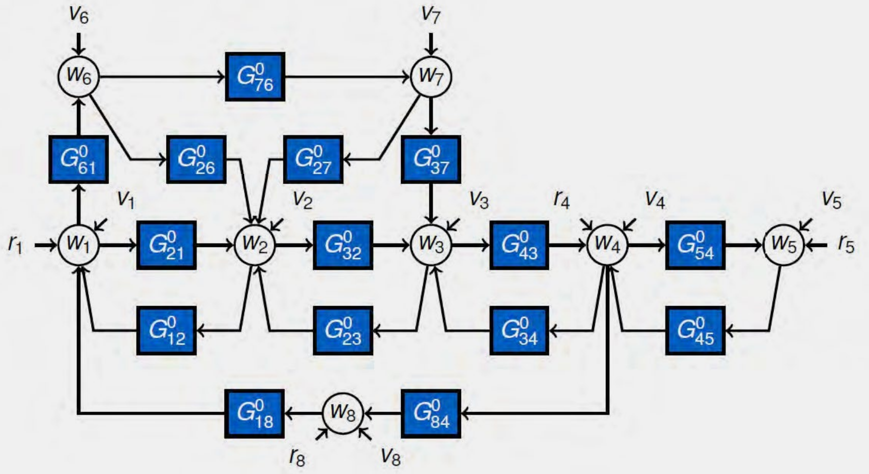
Dynamic network setup



Dynamic network setup

Assumptions:

- Known topology
- Known noise correlation structure
- Strictly proper modules
- Network is stable
- No sensor noise



$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} = \begin{bmatrix} 0 & G^0_{12} & \cdots & G^0_{1L} \\ G^0_{21} & 0 & \cdots & G^0_{2L} \\ \vdots & \ddots & \ddots & \vdots \\ G^0_{L1} & G^0_{L2} & \cdots & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix} + \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_L \end{bmatrix} + \underbrace{\begin{bmatrix} H^0_{11} & H^0_{12} & \cdots & H^0_{1L} \\ H^0_{21} & H^0_{22} & \cdots & H^0_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ H^0_{L1} & H^0_{L2} & \cdots & H^0_{LL} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_L \end{bmatrix}}_{v = H^0 e}$$

→ Dynamically correlated noise

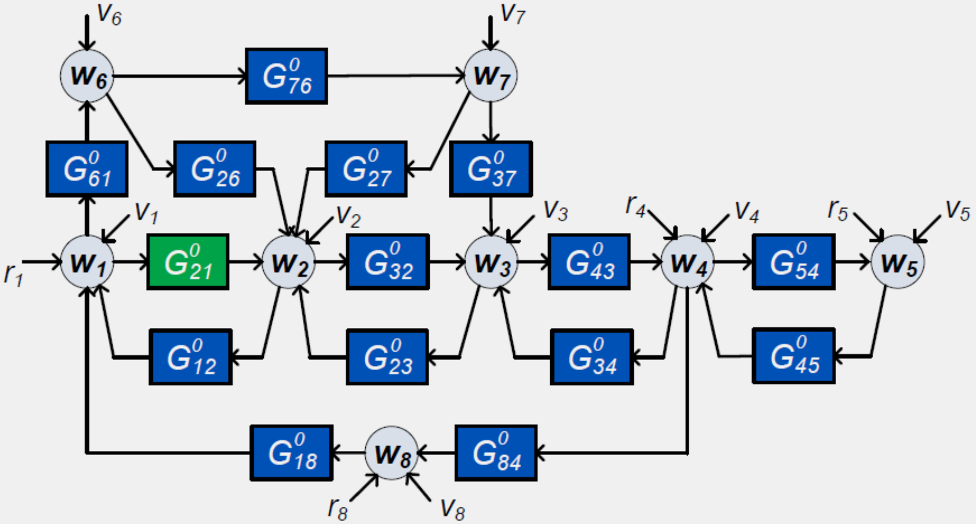
$$w = G^0(q)w + r + v \qquad\qquad w = (I - G^0)^{-1}(r + v)$$

P. M. J. Van den Hof, A. G. Dankers, P. S. C. Heuberger, and X. Bombois. Automatica, 49(10):2994–3006, 2013.

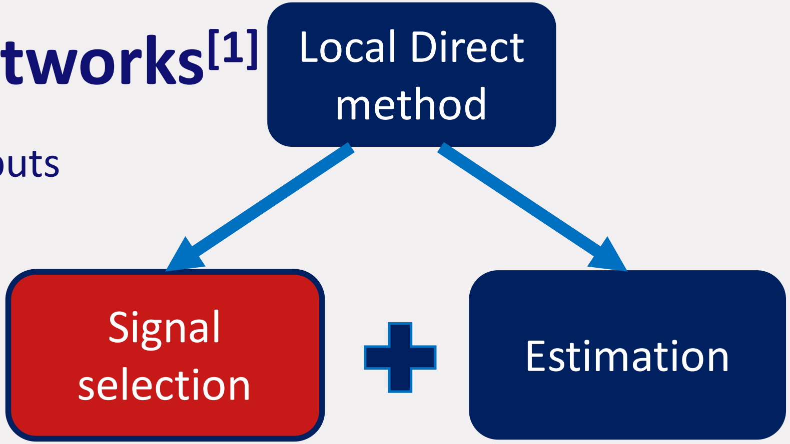
Local module identification

- For a network with known topology, identify a single module in a dynamic network based on the given data (w, r)

- For example, identify G_{21} on the basis of locally measured signals

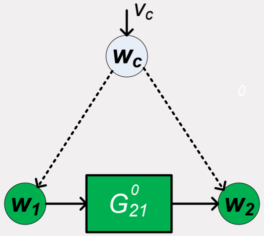


Local direct method (LDM) for networks[1]



- Choice of predictor model – with node signals as inputs

- Noise correlations and confounding variables are handled using MIMO noise model.



- Leading to a MIMO predictor model with common signals in inputs and outputs.

[1] K.R. Ramaswamy and P.M.J. Van den Hof., IEEE-TAC, 2021 (to appear)

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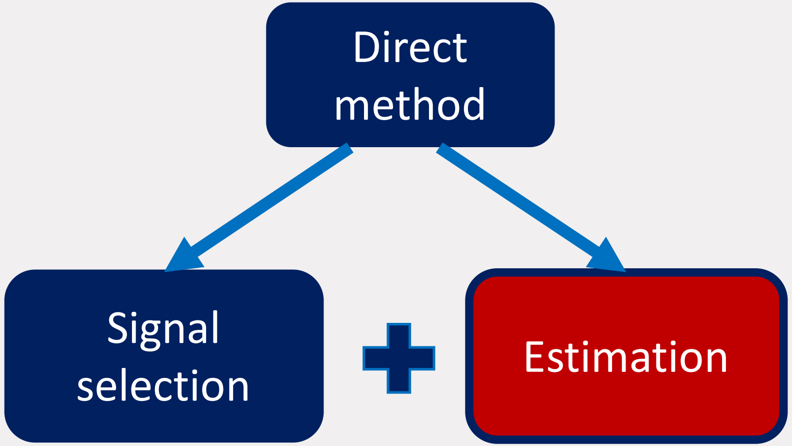
Problem ?



- Prediction error framework with prediction error:

$$\varepsilon(t,\theta) = \bar{H}(q,\theta)^{-1}[w_{\mathcal{Y}}(t) - \bar{G}(q,\theta)w_{\mathcal{D}}(t)]$$

- MIMO estimation with all modules parameterized.

- Brings in the following problems for large networks:
  - Model order selection step for each module
  - Large number of parameters to estimate
  - Algorithms to solve network MIMO estimation problem not available

[1] K.R. Ramaswamy and P.M.J. Van den Hof., IEEE-TAC, 2021 (to appear)

Approach

Predictor $$w_{\mathcal{Y}}(t) = (I - H^{-1})w_{\mathcal{Y}}(t) + H^{-1}Gw_{\mathcal{D}}(t) + e(t)$$

$$H^{-1}G = \begin{bmatrix} G_{ji} & 0 \\ 0 & 0 \end{bmatrix} + \boxed{S_G} \qquad I - H^{-1} = \boxed{S_H}$$

- Keep parametric model for target module G_{ji}
- How to model the other modules in the MIMO setup?

What we need?

Eliminate model order selection

Reduce number of parameters

Estimate the parameters

Modeling strategy

What we need?

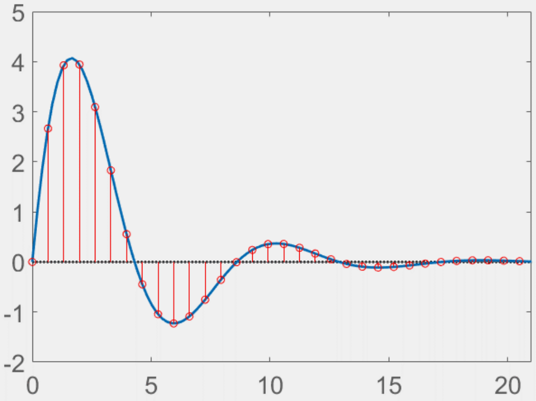
Eliminate model order selection → **Impulse response model (IRM)**

Reduce number of parameters → **IRM as Gaussian process**

Estimate the parameters → **Empirical Bayes approach**

Approach

Impulse response model (IRM) → IRM as Gaussian process → Empirical Bayes approach



$s \sim \mathcal{N}(0, \lambda K_\beta)$

Stable spline (SS) Kernel

$$[K_\beta]_{x,y} = \beta^{\max(x,y)}$$

$$\beta_j \in [0,1), \quad \lambda \geq 0$$

C. E. Rasmussen and C. K. I. Williams. Gaussian processes for machine learning. The MIT Press, 2006.

G. Pillonetto and G. De Nicolao. A new kernel-based approach for linear system identification. Automatica, 46(1), 2010.

Estimation of parameters

- Maximize the marginal pdf: $\hat{\eta} = \underset{\eta}{\operatorname{argmax}}\ p(w_{\mathcal{Y}}; \eta)$

- η contains parameters of target module, hyperparameters of GP and covariance of noise.

- We use Expectation – Maximization (EM) iterative algorithm to solve this.

  EM splits the maximization problem into simpler optimization problems

Noise covariance & hyperparameter λ → **Closed form solution**

Hyperparameter β → **Scalar optimization**

Target module parameters → **Nonlinear least squares problem**

P. Dempster, N. M. Laird, and D. B. Rubin. J. of the Royal Statistical Society. Series B (M.), 1977.

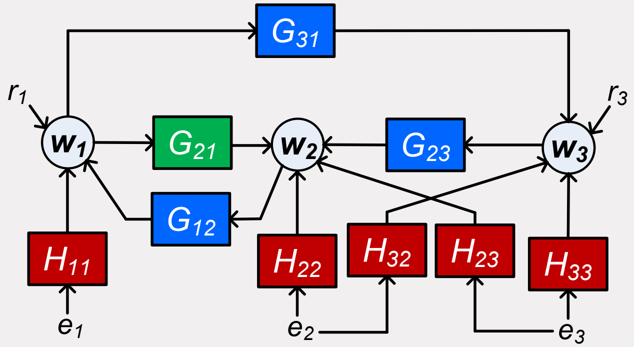
Numerical simulation

- Noise correlation between w_2 and w_3 handled by moving to output and (2 x 2) noise model[1]

- Signal selection (LDM): $\{w_1, w_3\} \rightarrow \{w_2, w_3\}$

- Data length = 500, MC simulations = 50

- We compare the developed EBLDM with:
  - Two-stage method with true order (TS+TO)
  - Direct method with true order (DM+TO) – $\{w_1, w_3\} \rightarrow \{w_2\}$
  - Direct method with model order selection (DM+MOS) – MISO setup



[1] K.R. Ramaswamy and P.M.J. Van den Hof., IEEE-TAC, 2021 (to appear)

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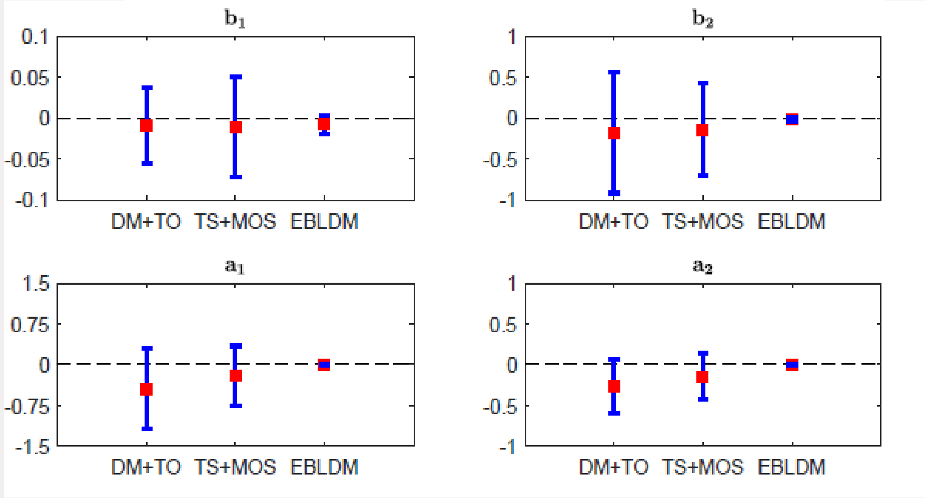
Numerical simulation

- EBDM → better fit than classical methods
- Gives smaller bias and reduced variance
- Reduction in variance → Due to regularization approach used in the method

$$\boxed{G_{31} = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}}}$$



Conclusion

- For correlated noise and large sized networks
  - number of parameters to estimate increases
  - model order selection step is computationally infeasible.
- An algorithm has been developed for correlated noise networks that can handle MIMO network identification
- No model order selection required and lesser number of parameters to estimate
- Reduced variance estimates attributed to the regularized kernel based methods

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