A regularized kernel-based method for learning a module in a dynamic network with correlated noise

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Dynamic network setup

\[ v_i \] node signal

\[ r_i \] external excitation

\[ v_i \] process noise

\[ G_{ij} \] modules
Dynamic network setup

Assumptions:

- Known topology
- Known noise correlation structure
- Strictly proper modules
- Network is stable
- No sensor noise

\[
\begin{bmatrix}
    w_1 \\
    w_2 \\
    \vdots \\
    w_L
\end{bmatrix} =
\begin{bmatrix}
    0 & G_{12}^0 & \cdots & G_{1L}^0 \\
    G_{21}^0 & 0 & \cdots & G_{2L}^0 \\
    \vdots & \vdots & \ddots & \vdots \\
    G_{L1}^0 & G_{L2}^0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
    w_1 \\
    w_2 \\
    \vdots \\
    w_L
\end{bmatrix} +
\begin{bmatrix}
    r_1 \\
    r_2 \\
    \vdots \\
    r_L
\end{bmatrix} +
\begin{bmatrix}
    H_{11}^0 & H_{12}^0 & \cdots & H_{1L}^0 \\
    H_{21}^0 & H_{22}^0 & \cdots & H_{2L}^0 \\
    \vdots & \vdots & \ddots & \vdots \\
    H_{L1}^0 & H_{L2}^0 & \cdots & H_{LL}^0
\end{bmatrix}
\begin{bmatrix}
    e_1 \\
    e_2 \\
    \vdots \\
    e_L
\end{bmatrix}
\]

\[
v = H^0 e
\]

\[
w = G^0(q)w + r + v
\]

\[
w = (I - G^0)^{-1}(r + v)
\]

Local module identification

- For a network with known topology, identify a single module in a dynamic network based on the given data \((w, r)\)

- For example, identify \(G_{21}\) on the basis of locally measured signals
Local direct method (LDM) for networks[1]

- Choice of predictor model – with node signals as inputs

- Noise correlations and confounding variables are handled using MIMO noise model.

- Leading to a MIMO predictor model with common signals in inputs and outputs.

Problem?

- Prediction error framework with prediction error:
  \[ \varepsilon(t, \theta) = \bar{H}(q, \theta)^{-1}[w_Y(t) - \bar{G}(q, \theta)w_D(t)] \]

- MIMO estimation with all modules parameterized.

- Brings in the following problems for large networks:
  - Model order selection step for each module
  - Large number of parameters to estimate
  - Algorithms to solve network MIMO estimation problem not available

**Approach**

**Predictor**

\[ w_Y(t) = (I - H^{-1})w_Y(t) + H^{-1}Gw_D(t) + e(t) \]

\[ H^{-1}G = \begin{bmatrix} G_{ji} & 0 \\ 0 & 0 \end{bmatrix} + S_G \]

\[ I - H^{-1} = S_H \]

- Keep parametric model for target module \( G_{ji} \)
- How to model the other modules in the MIMO setup?

**What we need?**

- Eliminate model order selection
- Reduce number of parameters
- Estimate the parameters
Modeling strategy

What we need?

- Eliminate model order selection
- Reduce number of parameters
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Approach

- Impulse response model (IRM)
- IRM as Gaussian process
- Empirical Bayes approach

Stable spline (SS) Kernel

\[ s \sim \mathcal{N}(0, \lambda K_\beta) \]

\[ [K_\beta]_{x,y} = \beta^{\text{max}}(x,y) \]

\[ \beta_j \in [0, 1), \quad \lambda \geq 0 \]


Estimation of parameters

- Maximize the marginal pdf: \( \hat{\eta} = \arg\max_{\eta} p(w; \eta) \)

- \( \eta \) contains parameters of target module, hyperparameters of GP and covariance of noise.

- We use Expectation – Maximization (EM) iterative algorithm to solve this.
  
  EM splits the maximization problem into simpler optimization problems:
  
  - Noise covariance & hyperparameter \( \lambda \) → Closed form solution
  - Hyperparameter \( \beta \) → Scalar optimization
  - Target module parameters → Nonlinear least squares problem

Numerical simulation

- Noise correlation between $w_2$ and $w_3$ handled by moving to output and (2 x 2) noise model\textsuperscript{[1]}

- Signal selection (LDM): \{w_1, w_3\} $\rightarrow$ \{w_2, w_3\}

- Data length = 500, MC simulations = 50

- We compare the developed EBLDM with:
  - Two-stage method with true order (TS+TO)
  - Direct method with true order (DM+TO) – \{w_1, w_3\} $\rightarrow$ \{w_2\}
  - Direct method with model order selection (DM+MOS) – MISO setup

Numerical simulation

- EBDM → better fit than classical methods
- Gives smaller bias and reduced variance
- Reduction in variance → Due to regularization approach used in the method

\[ G_{31} = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}} \]
Conclusion

- For correlated noise and large sized networks
  - number of parameters to estimate increases
  - model order selection step is computationally infeasible.

- An algorithm has been developed for correlated noise networks that can handle MIMO network identification

- No model order selection required and lesser number of parameters to estimate

- Reduced variance estimates attributed to the regularized kernel based methods
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