Identifiability in dynamic network identification

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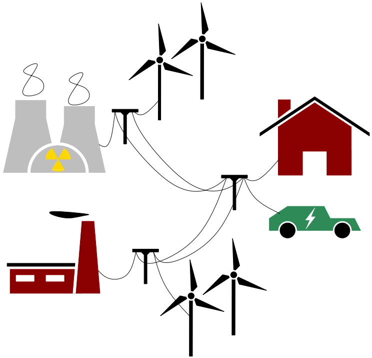
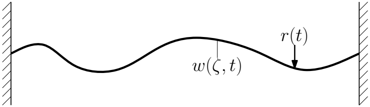
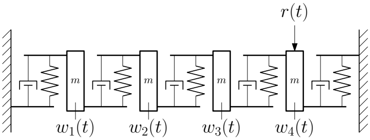
Dynamic networks appear in different domains







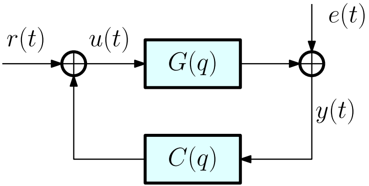
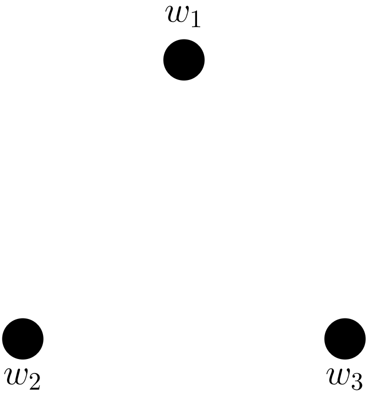


Illustration of the problem



- We have some measured node signals
- We want to identify the dynamics between the node signals

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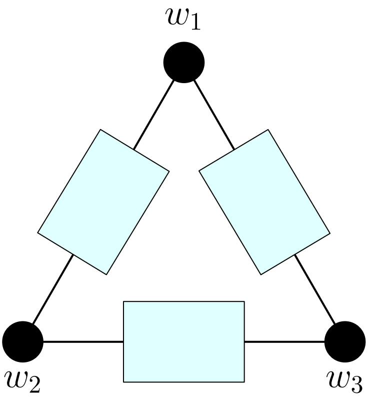

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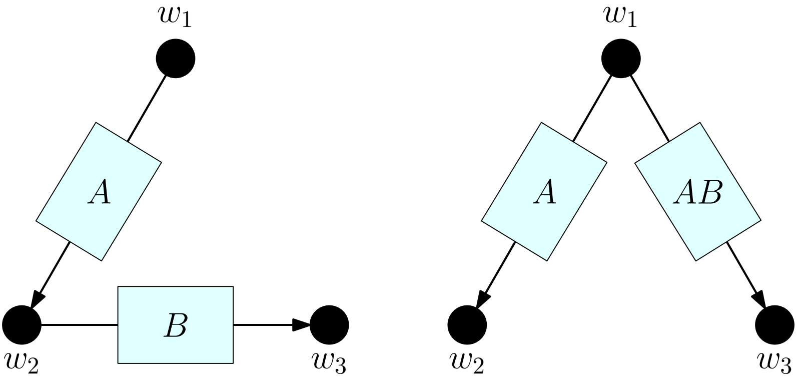
The identifiability question

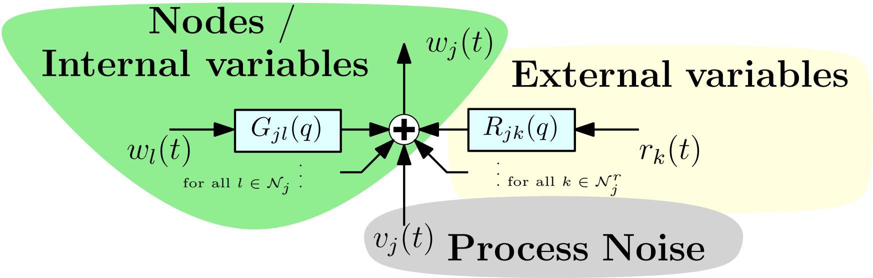


Can we distinguish between the networks??
Identifiability??

Outline

- Introduction
- Dynamic network setup
- Network predictor
- Global network identifiability
- Conclusions

Network definition



$$\underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix}}_{w(t)} = \underbrace{\begin{bmatrix} 0 & G_{12} & \cdots & G_{1L} \\ G_{21} & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & G_{L-1\ L} \\ G_{L1} & \cdots & G_{L\ L-1} & 0 \end{bmatrix}}_{G(q)} \underbrace{\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_L \end{bmatrix}}_{w(t)} + R \underbrace{\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_K \end{bmatrix}}_{r(t)} + \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix}}_{v(t)}$$

Typical assumptions in network identification literature

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix} = H(q) \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_L \end{bmatrix}}_{e(t)}, \text{ with } H(q) = \begin{bmatrix} H_{11}(q) & 0 & \cdots & 0 \\ 0 & H_{22}(q) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & H_{LL}(q) \end{bmatrix}$$

- Van den Hof et. al., Automatica, 2013
- Yuan et. al., Automatica, 2011
- Sanandaji et. al., ACC, 2011
- Materassi & Salapaka, IEEE trans AC, 2012

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Consequence: identification problem can be split into MISO problems

Our approach

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_L \end{bmatrix} = H(q) \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_L \end{bmatrix}}_{e(t)}, \text{ with } H(q) = \begin{bmatrix} H_{11}(q) & H_{12}(q) & \cdots & H_{1L}(q) \\ H_{21}(q) & H_{22}(q) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ H_{L1}(q) & \cdots & \cdots & H_{LL}(q) \end{bmatrix}$$

Noise contribution on all nodes can be correlated with each other

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Noise contribution on all nodes can be correlated with each other

Consequence: In the network identification problem all nodes should be treated symmetrically

The problem

- Identify the whole network, $G(q)$, $H(q)$ and $R(q)$, on the basis of a predictor for every node signal w_j.
- Formulate a condition, based on the network predictor, under which the networks can be distinguished from each other.
- The introduced identifiability notion is related to uniqueness of **dynamics** instead of parameters

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Problem statement

Under which condition is there a one-to-one relation between model dynamics and network predictor?

For standard configuration (open-loop, closed-loop) and for diagonal H the answer is relatively simple.

For non-diagonal H the answer is nontrivial!

Network predictor

$$w(t) = G(q)w(t) + R(q)r(t) + H(q)e(t)$$

Network predictor

$$w(t) = G(q)w(t) + R(q)r(t) + \left(H(q) - I\right)e(t) + e(t)$$

Network predictor

$$e(t) = H^{-1}(q)\left\{(I - G(q))^{-1}w(t) - R(q)r(t)\right\}$$

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$$w(t) = \underbrace{\left(I - H^{-1}(q)\right)}_{\text{Typical output filter}} w(t) + \underbrace{H^{-1}(q)G(q)}_{\text{New filter}}\, w(t) + \underbrace{H^{-1}(q)R(q)}_{\text{Typical input filter}} r(t) + e(t)$$

Typical structure results
from the network structure!

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$$\hat{w}(t|t-1) = \left\{I - H^{-1}(q)\,(I - G(q))\right\} w(t) + H^{-1}(q)R(q)r(t)$$

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Model structure

Define the network model structure:
$\mathcal{M}(\theta) = \{G(q,\theta),\ H(q,\theta),\ R(q,\theta)\}$

$$\hat{w}(t|t-1,\theta) = \underbrace{\left\{I - H^{-1}(q,\theta)\left(I - G(q,\theta)\right)\right\}}_{W_w(q,\theta)} w(t) + \underbrace{H^{-1}(q,\theta)R(q,\theta)}_{W_r(q,\theta)} r(t)$$

One-to-one relation

$$\hat{w}(t|t-1,\theta_1) = \hat{w}(t|t-1,\theta_2) \quad \textit{Predictor equality}$$

$$\theta_1 = \theta_2 \qquad \textit{Parameter equality}$$

One-to-one relation

$\hat{w}(t|t-1,\theta_1) = \hat{w}(t|t-1,\theta_2)$ *Predictor equality*

$\mathcal{M}(\theta_1) = \mathcal{M}(\theta_2)$ *Model equality*

$\Updownarrow_c$ Classical **identifiability**. [Ljung, 1999]

$\theta_1 = \theta_2$ *Parameter equality*

One-to-one relation

$$\hat{w}(t|t-1,\theta_1) = \hat{w}(t|t-1,\theta_2)$$

Predictor equality

$$\Updownarrow_c$$

Informative data [Ljung, 1999]

$\Phi_{w,r}(\omega) > 0$.

$$W_w(\theta_1) = W_w(\theta_2)$$
$$W_r(\theta_1) = W_r(\theta_2)$$

Predictor filter equality

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Model equality

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$$W_w = I - H^{-1}(I - G)$$
$$W_r = H^{-1}R$$

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$$\Updownarrow ???$$

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Predictor filter equality

$$\Updownarrow ???$$

Definition: Global network identifiability

$\mathcal{M}(\theta)$ is globally network identifiable when the implication holds in both directions.

$$\mathcal{M}(\theta_1) = \mathcal{M}(\theta_2)$$

Model equality

Global network identifiability

Proposition

$$\begin{array}{c} W_w(\theta_1) = W_w(\theta_2) \\ W_r(\theta_1) = W_r(\theta_2) \end{array} \Leftrightarrow T(\theta_1) = T(\theta_2)$$

$$T = (I - G)^{-1} \begin{bmatrix} H & R \end{bmatrix}$$

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Global network identifiability

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Theorem

$\mathcal{M}(\theta)$ is globally network identifiable if $\exists P(q)$ nonsingular, such that

$$\begin{bmatrix} H(q,\theta) & R(q,\theta) \end{bmatrix} P(q) = \begin{bmatrix} D(q,\theta) & F(q,\theta) \end{bmatrix}$$

with $D(q,\theta)$ diagonal, $\forall\, \theta$.

The condition is necessary when $G(q,\theta)$ is fully and independently parameterized.

Global network identifiability

Diagonal $H(q,\theta)$ trivially satisfies the theorem.

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Interpretation: *Every* node has excitation that enters *only* at that node

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What if we have a particular structure in G?

Flexibility of $G(q,\theta)$ can be reduced based on knowledge of the network.

Example: $G(q,\theta) = \begin{bmatrix} 0 & G_{12}(\theta) & 0 \\ G_{21}(\theta) & 0 & 0 \\ G_{31}(\theta) & G_{32}(\theta) & 0 \end{bmatrix}$

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Column 3 not parameterized, no need to excite node 3!

Conclusions

- New identifiability concept for dynamic networks.
- Identifiability split into two parts, dynamics and parameters
- Restrictions on $G(q, \theta)$, $H(q, \theta)$ and $R(q, \theta)$ can guarantee a one-to-one relation between $\mathcal{M}(\theta)$ and $\hat{w}(t|t-1, \theta)$.
- Ability to deal with correlated noise in the network.

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Thank you for the attention!!

Comments, questions and points of discussion are appreciated :)

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