Dutch Institute of Systems and Control

Course: System Identification
Fall 2020

Assignment number 2 (concerning Lecture 4)

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Giulio Bottegal
Department of Electrical Engineering
Eindhoven University of Technology
email: giulio.bottegal@gmail.com

Problem 1

Consider a FIR system described by its impulse response

$$g_0 = [g_0(1) \ \dots \ g_0(n)]^T,$$

and assume that N input-output samples are collected from the measurement model

$$y = Ug_0 + e, \quad e \sim \mathcal{N}(0, \sigma^2 I).$$

It has been shown that the optimal kernel – in terms of Mean Square Error (MSE) – is $K \propto g_0 g_0^T$.

a. Show that the regularization parameter is

$$\gamma = g_0^T U^T y - g_0^T U^T U g_0.$$

b. What is the corresponding MSE?

Problem 2

The choice of the input has a major impact on the performance of nonparametric identification techniques. Suppose that we want to identify the first $n = 100$ samples of the impulse response of a LTI system using LS/RLS. The system is fed by a stochastic signal that is obtained by filtering white noise through a 4th-order low-pass Butterworth filter.
Part 1

a. Generate input signals $u(t)$ for increasing values of the band of the Butterworth filter. Use the commands

```
[num_F,den_F] = butter(4,band);
u = filter(num_F,den_F,randn(N,1));
```

where `band` is in the interval $(0, 1]$ and $N = 1000$.

b. Inspect the condition number of the matrix $U^T U$ for increasing values of the band. What do you observe? What impact should you expect on an identified impulse response?

Part 2
Now, we suppose we use ridge-regression type RLS (i.e., $K = I_n$) to identify a system.

c. Would you expect an improvement in the quality of the estimated system? Why?

d. Derive the analytic relation between the condition number U^TU and the matrix $U^TU + \gamma I_n$ appearing in the expression of RLS.

Part 3
The James-Stein estimator is defined

$$\hat{g}_{\text{J-S}} = \left(1 - \frac{(n-2)\sigma^2}{\|U\hat{g}_{\text{LS}}\|^2}\right)\hat{g}_{\text{LS}}$$

An interesting property of this estimator is that it is *superefficient*, meaning that its MSE is always lower than the MSE of the least squares estimator.

e. Show that there exists a kernel $K_{\text{J-S}}$ and a regularization parameter $\gamma_{\text{J-S}}$ such that

$$\hat{g}_{\text{J-S}} = (U^TU + \gamma_{\text{J-S}}(K_{\text{J-S}})^{-1})^{-1}U^Ty.$$

f. Explain why in your opinion the James-Stein estimator does not constitute an attractive choice in system identification.