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Closed-loop identification of multivariable processes with part of the inputs controlled

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In many multivariable industrial processes a subset of the available input signals is being controlled. In this paper it is analysed in which sense the resulting partial closed-loop identification problem is actually a full closed-loop problem, or whether one can benefit from the presence of non-controlled inputs to simplify the identification problem. The analysis focuses on the bias properties of the plant estimate when applying the direct method of prediction error identification, and the possibilities to identify (parts of) the plant model without the need of simultaneously estimating full-order noise models.

1. Introduction

In the closed-loop identification literature, the experimental situation generally considered is the one depicted in figure 1; see Van den Hof (1998), Forssell and Ljung (1999) and Ljung (1999). However in industrial practice one will regularly encounter the situation as sketched in figure 2, where only subsets of the input and output signals are used in the control loop. Open-loop inputs might, for example, be either manipulated variables that are not manipulated by the controller or measurable disturbances. Open-loop outputs typically are variables of which measurements are available apart from the controlled variables (closed-loop outputs) and which one also would like to use as outputs of a model to be estimated. This latter situation frequently occurs at modern large scale industrial plants where the data acquisition systems typically deliver many more measurements of process variables than just the controlled variables.

The identification of partial closed-loop systems has not been dealt with extensively in the literature; although sometimes mentioned as, e.g., in Zhu (2001). For analysing the problem one could rephrase the partial closed-loop identification (PCLID) problem as a "complete" closed-loop identification (CCLID) problem where the controller has zero entries and some setpoint variables can not be excited. In this way the statistical properties of estimates can be analysed using existing theory on closed-loop prediction error identification.

It is well known that a closed-loop experimental situation has a severe impact on identification methods. When focussing on the so-called direct method (Ljung, 1999) of prediction error identification, two main consequences of the closed-loop situation are that

(a) a consistent plant model can only be identified if also the full noise model is estimated consistently; and
(b) the variance of the plant estimate is determined by only the noise-free part of the (closed-loop) input signals.

Point (a) can be rather problematic in situations with large numbers of inputs/outputs. Estimating a full-order plant and noise model can easily lead to high-dimensional and complex non-convex optimization problems that are hard to solve. As a result a separation of the identification problem can be attractive, where in a first step a plant model is identified and in a

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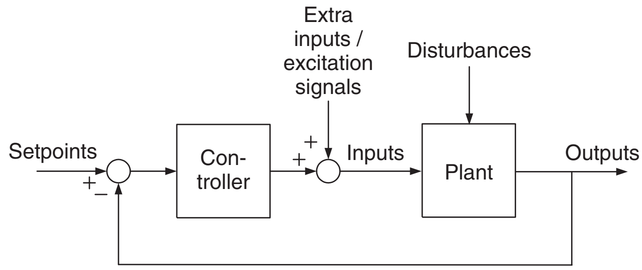

Figure 1. "Complete" closed-loop configuration.

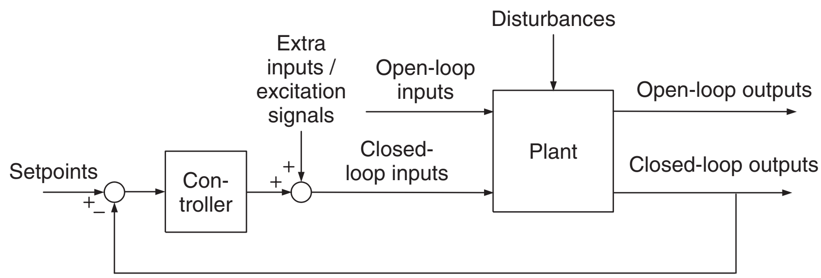

Figure 2. Partial closed-loop configuration.

second step the noise model is estimated if required, while both models can be validated separately. In an open-loop experimental setup this can be achieved by using independently parametrized plant and noise models. However in a closed-loop setting using the direct identification method an identification of the plant model separately will fail due to property (a) mentioned above.

In this paper the central question to be considered is: in the given situation of a partial closed-loop setting, is a separate identification of the plant model feasible, or in other words: can advantages of an open-loop experimental setup be used to facilitate separate identification of (possibly a part of) the plant model?

After specifying the appropriate setting and notation in § 2, the general convergence analysis for direct prediction error methods will be recalled in § 3. Next, in § 4 and §5, the particular situation of a partial closed-loop setting will be considered. In § 4, the most general situation, with both open-loop inputs and open-loop outputs being present, will be discussed. Section 5 will discuss the situations where either no open-loop inputs or no open-loop outputs are present. In § 6, simulation results are discussed that illustrate the presented theory. Section 7 will discuss some consequences of this theory and consider alternative closed-loop identification methods. The paper ends in § 8 with conclusions summarizing the answer to the question raised above.

2. Setup and notation

The closed-loop system configuration to be considered is sketched in figure 3, where $u_1(t)$ and $y_1(t)$ reflect the open-loop inputs and outputs, while $u_2(t)$ and $y_2(t)$ are the closed-loop (controlled) inputs and outputs. All indicated signals are considered to be multivariate. The system equations are given by

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = G_o(q)\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} + H_o(q)\begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \tag{1}$$

$$u_2(t) = K(q)[r(t) - y_2(t)], \tag{2}$$

where $r(t)$ is a set of setpoint signals and $K(q)$ a feedback controller. $H_o(q)$ is a monic stable and stably invertible noise filter, and $e(t) = [e_1^T(t)\ e_2^T(t)]^T$ a multivariate white noise process with covariance matrix $\mathbb{E}[e(t)e^T(t)] = \Lambda_0$.

The transfer function matrices corresponding to the plant and disturbance dynamics, $G_o(q)$ resp. $H_o(q)$, are partitioned according to

$$G_o(q) = \begin{bmatrix} G_o^{11}(q) & G_o^{12}(q) \\ G_o^{21}(q) & G_o^{22}(q) \end{bmatrix}; \quad H_o(q) = \begin{bmatrix} H_o^{11}(q) & H_o^{12}(q) \\ H_o^{21}(q) & H_o^{22}(q) \end{bmatrix}$$

with $G_o^{ji}(q)$ representing the part of $G_o(q)$ with $u_i(t)$ as its inputs and $y_j(t)$ as its outputs.

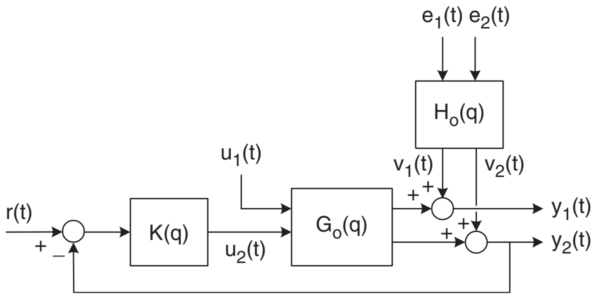

Figure 3. Partial closed-loop system (with both open- and closed-loop inputs and both open- and closed-loop outputs).

It is further assumed that possible excitation signals $u_1(t)$ are uncorrelated with $e(t)$, and that setpoint signals $r(t)$ are uncorrelated to $u_1(t)$ and $e(t)$.

In the direct method of prediction error identification a one-step ahead predictor model defined by $G(q,\theta)$ and $H(q,\theta)$ is considered, leading to a prediction error

$$\varepsilon(t,\theta) = H(q,\theta)^{-1}[y(t) - G(q,\theta)u(t)]$$

and an estimated model on the basis of N data is obtained by

$$\hat{\theta}_N = \arg\min_{\theta} \frac{1}{N}\sum_{t=1}^{N} \varepsilon^T(t,\theta)\Lambda^{-1}\varepsilon(t,\theta),$$

with Λ a symmetric positive definite weighting matrix. For further details and assumptions on the prediction error setting we refer to Ljung (1999), where it is shown that under fairly general conditions (the regularity conditions involve uniform stability of the model set, quasi-stationary of the input process, and a noise process having bounded fourth moment) the parameter estimate $\hat{\theta}_N$ converges as N tends to infinity with probability 1 to

$$D_c = \arg\min_{\theta} \bar{\mathbb{E}}\varepsilon^T(t,\theta)\Lambda^{-1}\varepsilon(t,\theta). \tag{3}$$

To simplify the notation in the remainder of this paper, the time t and shift operator q will be left out of it. For the same reason the notations $G_\theta = G(q,\theta)$, $H_\theta = H(q,\theta)$, etc. will be used

3. Convergence analysis of the direct approach

The asymptotic parameter estimate D_c (3) can be represented as a frequency domain integral by applying Parsseval's relation. For the considered closed-loop situation this results in (see Ljung (1999) for the scalar situation and Forsell and Ljung (1999) for the multivariable case)

$$D_c = \arg\min_{\theta} \int_{-\pi}^{\pi} tr\left[\left[(G_o - G_\theta)\ (H_o - H_\theta) \right] \Phi_{\chi_0} \times \begin{bmatrix} (G_o - G_\theta)^* \\ (H_o - H_\theta)^* \end{bmatrix} (H_\theta \Lambda H_\theta^*)^{-1} \right] d\omega, \tag{4}$$

where Φ_{χ_0} is the spectral density of the signal $\chi_0 := [u^T\ e^T]^T$, and $(\cdot)^*$ refers to the complex conjugate transpose.

In Corollary 5 of Forssell and Ljung (1999) the expression for D_c is reformulated into an expression that more directly represents the bias properties of the plant estimate $\hat{G}_\theta$. By writing

$$\Phi_{\chi_0} = \begin{pmatrix} I & 0 \\ \Phi_{eu}\Phi_u^{-1} & I \end{pmatrix} \begin{pmatrix} \Phi_u & 0 \\ 0 & \Phi_e^r \end{pmatrix} \begin{pmatrix} I & \Phi_u^{-1}\Phi_{ue} \\ 0 & I \end{pmatrix}$$

with $\Phi_e^r = \Lambda_o - \Phi_{eu}\Phi_u^{-1}\Phi_{ue}$, Forssell and Ljung (1999) show that under the additional assumption that u is persistently exciting (see Ljung (1999) for a definition), D_c is characterized by

$$D_c = \arg\min_{\theta} \int_{-\pi}^{\pi} tr[[(G_o + B_G - G_\theta)\Phi_u(G_o + B_G - G_\theta)^* + (H_o - H_\theta)\Phi_e^r(H_o - H_\theta)^*](H_\theta\Lambda H_\theta^*)^{-1}]d\omega \tag{5}$$

with

$$B_G = (H_o - H_\theta)\Phi_{eu}\Phi_u^{-1}. \tag{6}$$

The so called "bias-pull" B_G characterizes the amount of bias that is obtained for the G-estimate due to the controller induced correlation between the white noise terms e and inputs u. This bias-pull term might be considered as an extra bias on top of the bias introduced by the fact that the model structure G_θ might not be flexible enough to contain G_o ($G_o \notin \mathscr{G}$). It follows from the expression (5) that if $B_G = 0$, then $G_\theta = G_o$ is minimizing the trace expression, provided that the parameters that occur in H_θ are independent of the parameters in G_θ. In that situation the plant model will be identified without bias. Note that this situation typically occurs in a "full" open-loop problem where there is no correlation between noise signals and inputs. Then $\Phi_{eu} = 0$, and by (6) it follows that $B_G = 0$.

4. Convergence analysis in the case of PCLID data

4.1 *General case*

The central question considered in this paper is whether in a partial closed-loop setting a separate identification of the plant model and the noise model is possible with the direct identification approach. This question is addressed by verifying whether any of the elements of the bias pull B_G become structurally zero; the related entries of the G-estimate can, under some additional conditions to be discussed, be identified asymptotically unbiased (in case $G_o \in \mathcal{G}$) irrespective of the noise model. Hence, in order to be able to answer the central question considered here, one simply has to analyse the expression (6) for the particular partial closed-loop setting. In this section, this is done for the situation of both open-loop inputs and open-loop outputs being present. The situations where there are either no open-loop inputs or no open-loop outputs are discussed in § 5.

Because of the particular closed-loop configuration considered in the setup of figure 3 it follows that $\Phi_{e_1u_1} = 0$ and $\Phi_{e_2u_1} = 0$. As a result Φ_{eu} will be structured as

$$\Phi_{eu} = \begin{bmatrix} 0 & \star \\ 0 & \star \end{bmatrix},$$

where $\star$ refers to a general (non structural-zero) element. Substituting this into the expression (6), and taking into account that because of the closed-loop configuration Φ_u will be a matrix without structural-zero elements, there will not be an entry in B_G that is structurally equal to 0 (see equations (7) and (8)). This leads to the following proposition.

Proposition 1: *Consider the partial closed-loop identification problem as formulated above. In this situation the presence of an open-loop excitation signal u_1 does not imply that entries of the plant G_o can be identified asymptotically unbiased independent of the choice of the model structure for H_o.*

In other words: closing a single loop in an industrial process does generally turn the identification problem into a "full" closed-loop problem, and no single entries in G_o can be estimated asymptotically unbiased without fully parametrizing and identifying the noise models also.

In order to specify possible special cases the bias pull term B_G is specified in terms of its several entries. By simply analyzing the expression (6) (for $\Phi_{e_1u_1} = 0$ and $\Phi_{e_2u_1} = 0$) it follows that

$$B_G = \begin{pmatrix} B_G^{11} & B_G^{12} \\ B_G^{21} & B_G^{22} \end{pmatrix} \tag{7}$$

with

$$\left.\begin{aligned}
B_G^{11} &= -\big(H_o^{11} - H_\theta^{11}\big)\Phi_{e_1u_2}\Phi_{u_2}^{-1}\Phi_{u_2u_1}\Delta^{-1} \\
&\quad - \big(H_o^{12} - H_\theta^{12}\big)\Phi_{e_2u_2}\Phi_{u_2}^{-1}\Phi_{u_2u_1}\Delta^{-1} \\
B_G^{12} &= \big(H_o^{11} - H_\theta^{11}\big)\Phi_{e_1u_2}\big(\Phi_{u_2}^{-1} \\
&\quad + \Phi_{u_2}^{-1}\Phi_{u_2u_1}\Delta^{-1}\Phi_{u_1u_2}\Phi_{u_2}^{-1}\big) \\
&\quad + \big(H_o^{12} - H_\theta^{12}\big)\Phi_{e_2u_2}\big(\Phi_{u_2}^{-1} \\
&\quad + \Phi_{u_2}^{-1}\Phi_{u_2u_1}\Delta^{-1}\Phi_{u_1u_2}\Phi_{u_2}^{-1}\big) \\
B_G^{21} &= -\big(H_o^{21} - H_\theta^{21}\big)\Phi_{e_1u_2}\Phi_{u_2}^{-1}\Phi_{u_2u_1}\Delta^{-1} \\
&\quad - \big(H_o^{22} - H_\theta^{22}\big)\Phi_{e_2u_2}\Phi_{u_2}^{-1}\Phi_{u_2u_1}\Delta^{-1} \\
B_G^{22} &= \big(H_o^{21} - H_\theta^{21}\big)\Phi_{e_1u_2}\big(\Phi_{u_2}^{-1} \\
&\quad + \Phi_{u_2}^{-1}\Phi_{u_2u_1}\Delta^{-1}\Phi_{u_1u_2}\Phi_{u_2}^{-1}\big) \\
&\quad + \big(H_o^{22} - H_\theta^{22}\big)\Phi_{e_2u_2}\big(\Phi_{u_2}^{-1} \\
&\quad + \Phi_{u_2}^{-1}\Phi_{u_2u_1}\Delta^{-1}\Phi_{u_1u_2}\Phi_{u_2}^{-1}\big)
\end{aligned}\right\} \tag{8}$$

and $\Delta = \Phi_{u_1} - \Phi_{u_1u_2}\Phi_{u_2}^{-1}\Phi_{u_2u_1}$. Notice that none of the elements of B_G is zero, i.e., they all remain dependent on the bias of some part of the noise model: one might have expected that at least some part of B_G, e.g., B_G^{11} would have become zero. The fact that all elements of B_G remain non-zero immediately leads to the conclusions that, for this PCLID case, (i) the complete noise model must be estimated without bias in order to obtain a completely unbiased G-estimate (in case $G_o \in \mathcal{G}$) and (ii) no explicit user-defined tuning of any part of the bias of the G-estimate is possible. These conclusions are exactly the same as for the CCLID case and, thus, this PCLID problem should be treated as a CCLID problem (or one should resort to an alternative PCLID method; see § 7).

The expressions given above for the bias-pull terms are valid for the most general situation possible, i.e., without any additional structural conditions on G_o and/or H_o. In the next two subsections two different special cases will be considered.

4.2 *The case of uncorrelated disturbances v_1 and v_2*

A particular case that leads to special results is when the disturbances v_1 acting on the open-loop outputs are uncorrelated with those acting on the closed-loop outputs v_2. This can be represented by the requirements that $H_o^{21} = 0$, $H_o^{12} = 0$, and Λ_o block-diagonal. The direct consequence then is that $\Phi_{e_1u_2} = 0$ and this can further simplify the expressions for B_G, as formulated next.

Proposition 2: *Consider the partial closed-loop identification problem as formulated before. Under the additional conditions:*

(i) *v_1 and v_2 are uncorrelated, and*
(ii) *the model structure used for identification satisfies $H_\theta^{12} = 0$*

B_G will satisfy

$$B_G = \begin{bmatrix} 0 & 0 \\ \star & \star \end{bmatrix}.$$

As a result the plant transfers G_o^{11} and G_o^{12} can be identified asymptotically unbiased, irrespective of the noise model H_θ, provided that

- *u is persistently exciting, and*
- *the parameters of G_θ^{11} and G_θ^{12} are independent of the parameters in the remaining transfers of G_θ and H_θ.*

The proposition shows that when the output disturbances on the two different types of outputs are uncorrelated, the entries in G_o related to the open-loop output y_1 can be identified in an unbiased way, irrespective of the noise model. To this end the two inputs u_1 and u_2 need to be considered jointly. One can not retain the same properties of unbiasedness if simply u_1 and y_1 are taken to identify the transfer G_θ^{11} separately.

The restriction on the parametrizations that is formulated in Proposition 2 implies that there will occur problems if a multivariable parametrization for G_θ is used in which coupling of parameters in several entries of the transfer matrix occur. The entries that will be identified asymptotically unbiased need to be parametrized independent of the parameters in the other transfer entries of G_θ. Attractive parametrizations that allow independent parametrizations in the transfer entries to different output signals are e.g. finite impulse response models, models based on orthogonal basis function expansions (Ninness *et al*. 1995, Van den Hof *et al*. 1995), and state space models in output companion forms (Gevers and Wertz 1984). Less attractive model structures are general state space models, as e.g., used in subspace identification (Van Overschee and de Moor 1996), and multivariable polynomial models as, e.g., ARX models (Ljung 1999).

4.3 *The case of no cross-coupling: $G_o^{21} = 0$*

If the plant's transfer from open-loop inputs u_1 to closed-loop outputs y_2 is known to be 0, a situation results where $\Phi_{u_2u_1} = 0$ and consequently Φ_u becomes block diagonal. The situation is rather restrictive, but is still special enough to be considered separately. Substituting $\Phi_{u_2u_1} = 0$ into the expressions for B_G it follows that

$$B_G = \begin{bmatrix} 0 & \star \\ 0 & \star \end{bmatrix}.$$

As a result the plant transfers G_o^{11} and G_o^{21} can be identified asymptotically unbiased, irrespective of the noise model H_θ, under conditions that are similar as formulated in Proposition 2. Note that the situation of the entry G_o^{21} now is trivial, as it is presumed to be 0!

This situation allows for a separation of the identification problem. By only considering the measurements u_1 and y_1, the transfer function G_o^{11} can be identified unbiasedly (irrespective of H_θ) even when discarding the effect of u_2. Discarding the effect of u_2, i.e., discarding the transfer G_o^{12}, then leads to an increase of the variance of the estimate, but not to a bias.

If both the condition $G_o^{21} = 0$ and the set of assumptions from the previous subsection are satisfied, the resulting structure for B_G is

$$B_G = \begin{bmatrix} 0 & 0 \\ 0 & \star \end{bmatrix}.$$

As a result unbiased estimates (irrespective of H_θ) can be obtained for G_θ^{11}, G_θ^{12} and G_θ^{21}, provided that these model entries are parametrized independent of the remaining entry in G_θ and from all entries in H_θ. An unbiased estimate for G_θ^{22} can (again) only be obtained if an unbiased estimate is obtained of the noise model H_θ^{22}.

5. The PCLID problem with only controlled inputs or outputs

In this section the same line of analysis will be followed to investigate and discuss, briefly, convergence of the direct approach when applied to the PCLID situations where either no open-loop inputs or no open-loop outputs are present. It will become evident that similar conclusions as stated in the previous section are valid also for these PCLID situations.

5.1 *The case of no open-loop inputs: $dim(u_1) = 0$*

In this PCLID case the bias-pull term can be analysed similarly as in §4.1, leading to

$$B_G = \begin{pmatrix} B_G^{12} \\ B_G^{22} \end{pmatrix} \tag{9}$$

with

$$\left.\begin{aligned} B_G^{12} &= \left(H_o^{11} - H_\theta^{11}\right)\Phi_{e_1u_2}\Phi_{u_2}^{-1} + \left(H_o^{12} - H_\theta^{12}\right)\Phi_{e_2u_2}\Phi_{u_2}^{-1} \\ B_G^{22} &= \left(H_o^{21} - H_\theta^{21}\right)\Phi_{e_1u_2}\Phi_{u_2}^{-1} + \left(H_o^{22} - H_\theta^{22}\right)\Phi_{e_2u_2}\Phi_{u_2}^{-1} \end{aligned}\right\} \tag{10}$$

Note that these expressions also follow from (8) by setting $\Phi_{u_1u_2} = \Phi_{u_2u_1} = 0$. Also here, as can be seen, none of the bias-pull terms becomes independent of the biases of the noise model estimates. Hence, also this PCLID problem should basically be treated as a full closed-loop identification problem meaning that a completely unbiased H-estimate must be obtained in order to obtain a completely unbiased G-estimate.

Under the additional assumptions of Proposition 2 (disturbances v_1 and v_2 uncorrelated, etc.)

$$B_G = \begin{pmatrix} 0 \\ \left(H_o^{22} - H_\theta^{22}\right)\Phi_{e_2u_2}\Phi_{u_2}^{-1} \end{pmatrix} \tag{11}$$

As a result, an unbiased estimate of G_o^{12} can be obtained irrespective of H_θ under the usual conditions of independent parametrization. The transfer G_o^{22} can only be estimated unbiasedly, if the noise model H_θ^{22} is estimated without bias.

5.2 *The case of no open-loop outputs: dim*$(y_1) = 0$

If y_1 is not present, the bias-pull term reduces to

$$B_G = \begin{pmatrix} B_G^{21} & B_G^{22} \end{pmatrix} \tag{12}$$

with

$$\left.\begin{aligned} B_G^{21} &= -\left(H_o^{22} - H_\theta^{22}\right)\Phi_{e_2u_2}\Phi_{u_2}^{-1}\Phi_{u_2u_1}\Delta^{-1} \\ B_G^{22} &= \left(H_o^{22} - H_\theta^{22}\right)\Phi_{e_2u_2}\Phi_{u_2}^{-1} \\ &\quad \times \left(I_{nu_2} + \Phi_{u_2u_1}\Delta^{-1}\Phi_{u_1u_2}\Phi_{u_2}^{-1}\right) \end{aligned}\right\} \tag{13}$$

and Δ as given before. Since B_G does not contain any structural zeros, the PCLID problem basically should, again, be treated as a full closed-loop identification problem.

The additional (and simplifying) assumption that $B_G^{21} = 0$, implying that $\Phi_{u_2u_1} = 0$, does not lead to any apparent advantage as it only affects B_G^{21} which is zero by assumption in this case.

6. Simulation example

In order to illustrate the theoretical results that are discussed in this paper, an example is presented involving simulations with a linear time-invariant process G_o with 3 inputs and 2 outputs, in a configuration as sketched in figure 3, where all signals are scalar-valued, except for the open-loop input u_1 which is two-dimensional. In other words, in the considered simulations a scalar-valued feedback system is applied to a 3 input, 2 output process.

The process models G_o that are used in the simulations (full and with $G_{21} = 0$) were, in fact, derived from a model that was identified from real-life data obtained, in a partial closed-loop setting, from a large-scale municipal solid waste combustion (MSWC) plant. Similarly, the (full and diagonal) disturbance models H_o that are used in the examples were derived from a disturbance model that was obtained from the same real-life MSWC plant data. Actually, this MSWC plant PCLID problem formed the motivation for the work presented here. The MSWC model was estimated in a similar fashion as described in Leskens *et al.* (2002), i.e., in the two stage manner of Van den Hof and Schrama (1993) (see the discussion in § 7) and using a high order modelling and model reduction approach. The disturbance model was estimated according to the two step procedure discussed at the beginning of § 7. The specifics of the identification of the MSWC plant will not be discussed in this paper as it is outside its scope.

In all identification experiments to be discussed the closed-loop system is excited by the two open-loop input signals and the external reference signal r, all chosen as uncorrelated white noise processes with (the almost asymptotic) length $N = 120\,000$. This experiment length was chosen this (very) large in order to sufficiently minimize any accidental (variance) error in the estimated model(s), thereby properly disclosing any bias error.

6.1 *Output error and Box Jenkins models – General case*

In order to verify whether the process dynamics can be identified without modelling the noise dynamics, a full-order output error (OE) model is identified (situation $G_o \in \mathcal{G}$). The results of this identification are shown in figure 4, where the step response of G_o is compared with 5 realizations of estimated output error models. It is apparent that all entries of the identified model show a bias (on top of a relatively small variance error). The results are compared with 5 realizations of estimated Box–Jenkins (BJ) models that include a noise model (situation $S \in \mathcal{M}$), that clearly do not exhibit bias. This illustrates the results of Proposition 1: once single feedback is applied to a system, output error models no longer provide asymptotically unbiased process models.

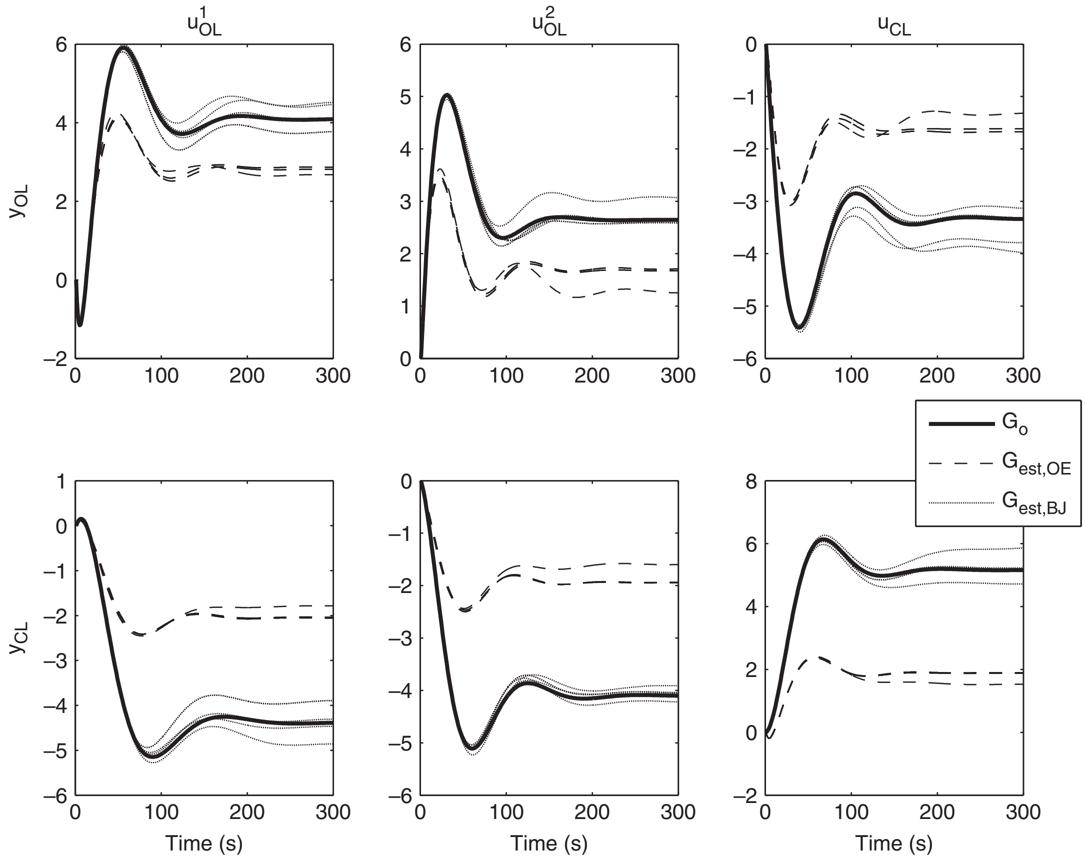

Figure 4. Step responses of G_o (solid), five realizations of OE estimates (dashed) and of BJ estimates (dotted) for data generating system with full noise model H_o.

6.2 *Output error models – Situation of diagonal H_o*

In order to illustrate the results of Proposition 2, the noise process is chosen to be diagonal. For this situation output error models are identified (situation $G_o \in \mathscr{G}$), where the entries of the process models are parametrized independently. In figure 5 the step responses of five realizations of estimated output error models are shown and compared to the process G_o. It appears, as predicted by Proposition 2, that the entries related to the open-loop output y_1, i.e., the first row of transfers, are unbiased, whereas the identified transfers towards the closed-loop output y_2 are biased.

6.3 *Output error models – Situation of $G_{21} = 0$*

The result of § 4.3 is illustrated by returning to the original (full) noise dynamics H_o but by setting the cross-coupling term in the process dynamics $G_o^{21} = 0$. Figure 6 shows the results of, again, 5 estimated output error models where the entries of the process models are parametrized independently. In this situation the transfers related to the open-loop inputs (u_1) are asymptotically unbiased. This appears in the first two column entries of the models.

7. Discussion of the results and alternative methods

The results presented in the previous sections point to limited possibilities for the direct PE identification method to partition a partial closed-loop identification problem into several subsequent steps. Such a partitioning can be very attractive in open-loop problems. In multivariable open-loop identification problems (cf, figure 7) it is generally possible to perform the following subsequent steps:

- first identify a consistent plant model $\hat{G}$, and validate this model;
- next (if necessary) identify an accurate noise model $\hat{H}$.

The separation of these steps is attractive from a computational point of view, but also in view of a separate order and structure determination (validation) of the model transfers $\hat{G}$ and $\hat{H}$. The first step in this procedure can even be partitioned in separate experiments, where

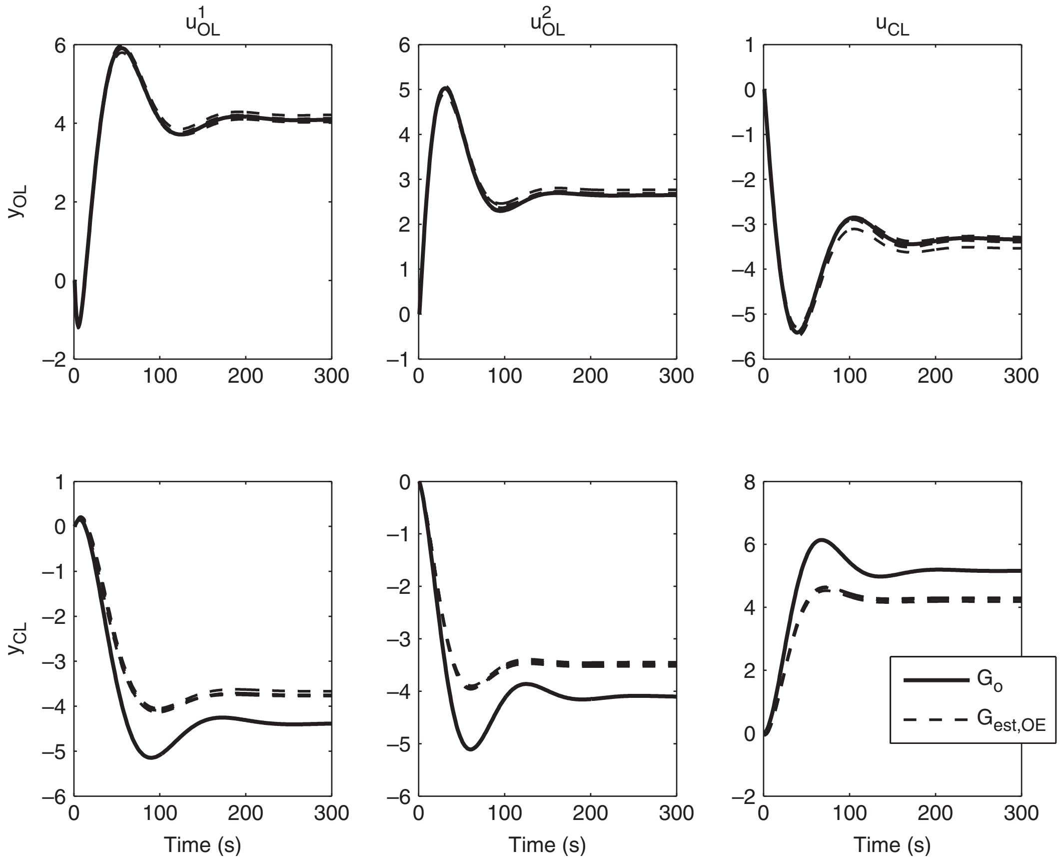

Figure 5. Step responses of G_o (solid) and five realizations of OE estimates (dashed) for data generating system with block diagonal noise model H_o.

one input signal at a time is excited, and corresponding SIMO models are identified. If the input signals are uncorrelated, i.e., Φ_u is diagonal, the separation into SIMO identification problems can even be made on the basis of one dataset where all inputs are excited simultaneously. In all these situations there will be no bias in the plant estimate $\hat{G}$. Although care has to be taken when validating process models without the availability of accurate noise models (see, e.g., Douma *et al.* (2005)) the separation of identifying process and noise models has several advantages.

For the partial closed-loop identification problems as sketched and discussed in the previous sections, it appears that all these properties are lost when using the direct closed-loop identification method, once one single loop around the system is closed. Identification of an unbiased $\hat{G}$ generally requires a full identification of the noise model $\hat{H}$.

Only in special cases (v_1 and v_2 uncorrelated) a part of G_o can be estimated unbiased without any limiting conditions on the estimated noise model. For the remaining part of the plant model at least part of the noise model needs to be identified simultaneously.

Separation of the multivariable experiments into single input excitations (and SIMO model identifications) will always lead to biased models, because of the fact that input signals will be correlated through the presence of feedback.

As an alternative for the rather pessimistic results on closed-loop identification with the direct prediction error method, indirect methods (Van den Hof and De Callafon 1996) or joint input-output methods as, e.g., the two-stage method (Van den Hof and Schrama 1993) or projection method (Forssell and Ljung 1999) can be considered. In the two-stage/projection approach the transfer function from reference input to closed-loop input is estimated first, and this (unbiased) model estimate is used to construct a filtered closed-loop input in which the noise-dependent part of the signal is removed. In the second stage the plant model is then estimated on the basis of the reconstructed input signal and the measured output. When applying the two-stage method to the PCLID problem, the first stage consists of estimating the transfer from both r and u_1 to u_2 and subsequently constructing the noise free part $\hat{u}_2$ of the latter signal(s). In the second stage, the plant and

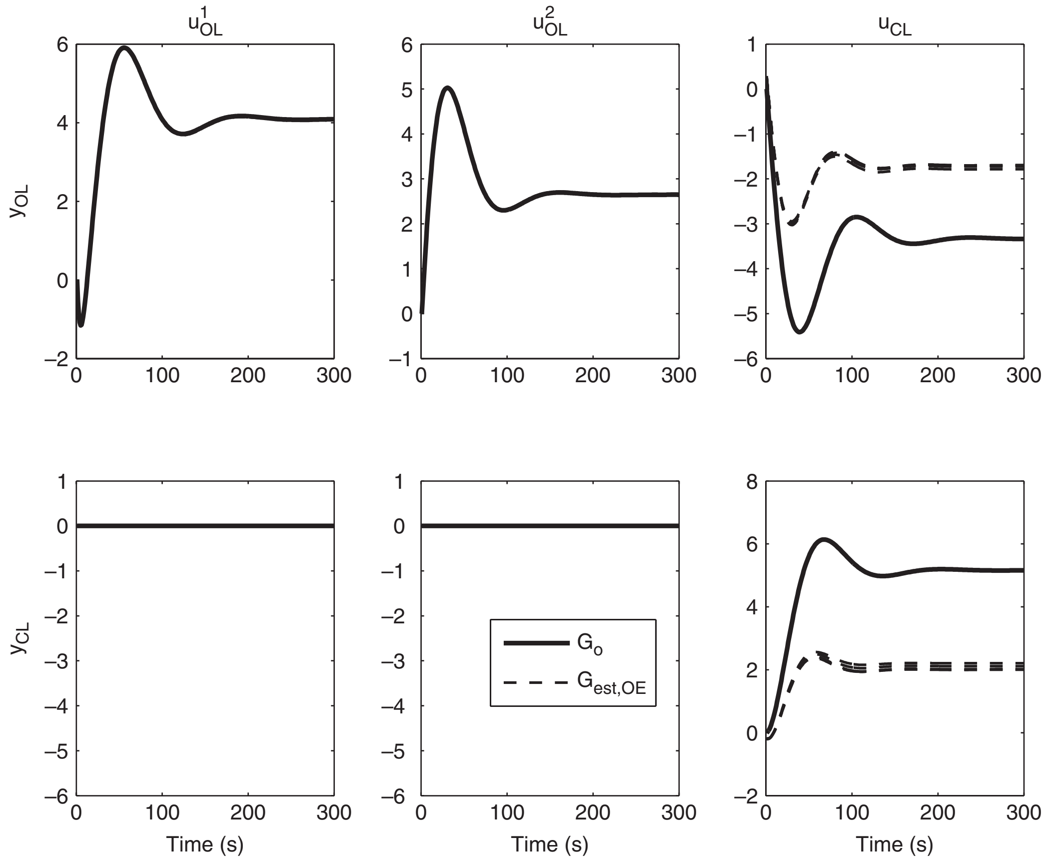

Figure 6. Step responses of G_o (solid) and five realizations of OE estimates (dashed) for data generating system with full noise model and $G_{21} = 0$.

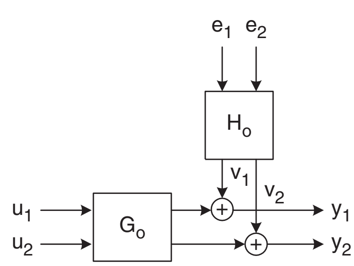

Figure 7. Multivariable open-loop configuration.

(if necessary) noise model can then be obtained unbiasedly and in separate steps via estimating the transfer from $\hat{u} := [u_1^T\ \hat{u}_2^T]^T$ to the outputs y. Note that it is important to also include u_1 in the first step, in order to avoid that its effect is being considered as an unmeasured disturbance, leading to increased variance of the estimated model.

Although the direct prediction error method is attractive from a statistical efficiency point of view, alternative indirect (or joint i/o) methods can have particular advantages as indicated here.

The current paper focusses only on bias-properties of estimated models. Variance properties will have to be complemented, and will contribute to providing answers to the question how to design the cheapest multivariable experiments (in terms of plant excitation and experiment time) in order to guarantee identified models within a prespecified bound of uncertainty, as addressed, e.g., for SISO processes in Bombois *et al*. (2006).

8. Conclusions

In this paper, it has been shown that for the direct method of (closed-loop) prediction error identification in general any partial closed-loop identification (PCLID) problem has basically the same characteristics as a full closed-loop identification problem and should therefore be treated as such.

This implies that also for PCLID problems all transfer functions of the noise model must be estimated without bias for all transfer functions of the G-estimate to be unbiased. Only in the special case that the output disturbances on open-loop and closed-loop outputs are uncorrelated, parts of the plant model can be identified unbiased without identifying a noise model.

The implication of these results is that the option to partition the (large scale) identification problem into subsequent steps (first identifying G_o and subsequently H_o) is not feasible for this approach, nor is it possible to partition the MIMO identification problem into independent SIMO problems.

It has been illustrated that these latter problems can be overcome by other methods of closed-loop identification as, e.g., the two-stage method. Especially in multivariable problems with only a limited number of loops closed, it can be very attractive to remove the noise influence on the closed-loop inputs in a first step, and reformulate the identification as an open-loop problem, retaining all favorable properties of open-loop identification methods.

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