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Frequency domain identification of passive local modules in linear dynamic networks☆

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ABSTRACT

We develop a novel frequency-domain approach to address the important open issue of estimating passive local modules within dynamic networks. The method applies an approach based on two stages, a non-parametric and a parametric one. The parametric stage is an extension of the vector fitting technique that incorporates energy consistency conditions as a fundamental component of the identification procedure, forming a path of the passive model in the Sanathanan–Koerner iterations. The approach includes a formulation via linear matrix inequalities to enforce energy-balance conditions resulting in a convex optimization problem. The approach is practical even under weak assumptions on noise, enabling real-world applications. Numerical simulations illustrate the potential of the developed method to effectively estimate local passive modules in dynamic networks.

1. Introduction

Increasing complexity and interconnectivity of systems is an evident trend in many domains. Autonomous driving systems and decentralized control systems serve as illustrative examples, which rely on complex networks of sensors, processors and actuators that act concertedly to control and respond to changing conditions. A distinguishing feature of these systems is that they form dynamic networks which are dynamical systems intricately interconnected. In addition, these structured interconnections are driven by and subjected to exogenous excitations and disturbances. As in [1], dynamic networks can be defined as collection of internal variables mutually and dynamically related (named modules) in a network.

While identifying isolated systems has become mainstream within the system identification community, the identification of modules intricately interconnected remains a challenging problem given the presence of highly correlated signals that affect the measurements [2,3]. The increased complexity of dynamic networks renders impractical traditional methods for obtaining a single model to represent local module inside a network.

Identification of dynamic networks is usually categorized into full network identification and local network identification, see [1]. In any case, network topology is assumed known, i.e., all interconnections between modules are given as problem data. While the full network identification strategy focuses on obtaining the overall dynamic behavior of the system, local identification strategy focuses on estimating only local dynamics, e.g. [1]. The problem of network topology identification has become a separate research area.

Local identification of dynamic networks entails obtaining a consistent estimate of (one or more) target modules. It can be achieved by adjusting existing closed-loop methods [1,2,4,5]. A description on how to adjust Predictor Error Method (PEM) to suit the needs of dynamic networks can be found in [1,2]. To give an instance of such adjustments, direct methods for module identification modify a one-step ahead predictor to identify modules in networks whereas indirect two-stages methods decorrelate signals before running a predictor [6].

Passivity of systems and networks is a fundamental property which may be defined in terms of energy dissipation and transformation. A physical system is denoted as passive when it is unable to generate energy [7]. Passive modeling has become pervasive across a number of research areas from network [8] to control [9] systems.

An important open issue within local network identification is the requirement that the estimated model of a passive local module within

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a dynamic network is also passive. Passivity possesses a closure property, which means that any interconnection of passive systems leads to an overall passive system [10]. This mathematical property is particularly suited to the context of dynamic networks, which involves extensive and intricate interconnections of subsystems. Last but not least, models must reflect fundamental physical properties of the system being represented. Consequently, if a dynamic network is formed out of passive systems, each model of these systems must be passive.

Within system identification, passivity has been extensively explored in the literature [10–16] and it is well known that system identification methods usually do not guarantee a passive estimate even if the original system is passive [11,13,15,17]. To tackle this issue, the so-called passivity enforcement algorithms have been proposed to enforce passivity as post-processing procedure [17].

Nonetheless, despite being explored in the literature on SISO/MIMO system identification [10–16], this passivity enforcement issue has remained unaddressed in the context of dynamic networks, which is the focus of this paper.

As a result, we present a frequency-domain approach that identifies passive local models in dynamic networks by incorporating Positive-Real Lemma (PRL) constraints into the estimation process. The inclusion of energy balance conditions into the estimation process improves reliability while making it suitable for dynamic network applications. This new method consists of the following stages:

(1) Obtaining a Frequency Response Function (FRF) for a local module(s) of interest using the indirect Local Polynomial Method (iLPM) [18];
(2) Deriving a state-space realization for this FRF via an extension of the Vector Fitting (VF) method that directly estimates a passive model.

This paper proposes a novel methodology for this second stage, termed here by Passive Vector Fitting (PVF). It constructs a sequence of passive models during the Sanathanan–Koerner iterations, ensuring that the final estimated model is inherently passive. This characteristic makes the method different from conventional VF, which relies on post-processing techniques applied to an estimated non-passive model.

This innovation brings the guarantee of passivity into the realm of local module identification for dynamic networks, addressing an open question in this field. The PVF algorithm estimates a parametric model for target local module from network data.

This paper is structured into 6 sections. In Section 2, we lay out the problem statement. Section 3 contains the formulation of energy balance conditions for discrete-time systems. Section 4 presents the frequency domain identification method that estimates a passive realization. Numerical simulations are then provided in Section 5, where the method is validated. Lastly, conclusions are presented in Section 6.

We use the following notation throughout the paper: G^0 is used to denote a transfer function for data-generating system target module with G its corresponding frequency response data. $\widetilde{G}$ denotes its frequency response function estimate. $\breve{G}$ denotes a passive state-space realization of the target module. The variable ω refers to the frequency vector, while w represents the network's node signals.

2. Problem statement

A general discrete-time model for a dynamic networks is now discussed [2]. Assuming a network with L nodes then, to each node is associated internal variables $w_j(t)$, $j = 1, \ldots, L$. A module $G^0_{ji}(q)$ is the dynamic between node i to j and is defined as a rational proper transfer function which admits an equivalent state-space realization with $i = 1, \ldots, L$. Additionally, any external manipulated excitation variable to node j is denoted $r_j(t)$ and $v_j(t)$ stands for process noise at the same node, that is:

$$w_j(t) = \sum_{i \in \mathcal{N}_j} G^0_{ji}(q) w_i(t) + r_j(t) + v_j(t), \tag{1}$$

with q^{-1} denoting the delay operator, i.e., $q^{-1}u_j(t) = u_j(t-1)$. Dynamic networks are herein considered without self loops, meaning $G^0_{jj} = 0$. The set $\mathcal{N}_j$ denotes the indices of node signals w_i connected to w_j, $i \neq j$, i.e., no zero modules G^0_{ji}. A zero module refers to a transfer function G^0_{ji} that is identically zero, indicating that there is no dynamic relationship from node i to node j. The set $\mathcal{R}$ denotes the indices of non-zero external signals r_j, $j = 1, \ldots, L$. Furthermore, the process noise can be given by:

$$v_j(t) = \sum_{j \in \mathcal{E}_j} H_j(q) e_j(t). \tag{2}$$

in which e_j is a zero-mean white noise process. $\mathcal{E}_j$ denotes the set of indices of white noise source signals e_j. $H_j(q)$ being a proper transfer function, monic, stable, minimum-phase. Each module G^0_{ji} can be conveniently represented as a minimal linear discrete-time state-space realization:

$$\begin{aligned} x(t+1) &= \mathbf{A}x(t) + \mathbf{b}w_i(t), \\ w_j(t) &= \mathbf{c}x(t) + c_0 w_i(t) + r_j(t) + v_j(t). \end{aligned} \tag{3}$$

with $x(t) \in \mathbb{R}^n$, $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{b} \in \mathbb{R}^{n\times 1}$, $\mathbf{c} \in \mathbb{R}^{1\times n}$ and $c_0 \in \mathbb{R}$.

Eq. (1) can also be expressed in matrix form with the time and delay indices omitted for convenience:

$$\mathbf{w} = \mathbf{G}\mathbf{w} + \mathbf{r} + \mathbf{v}. \tag{4}$$

where $\mathbf{G} = \begin{bmatrix} 0 & G_{12}(q) & \cdots & G_{1L}(q) \\ G_{21}(q) & 0 & \cdots & G_{2L}(q) \\ \vdots & \ddots & \ddots & \vdots \\ G_{L1}(q) & G_{L2}(q) & \cdots & 0 \end{bmatrix}$ is a square matrix of dimensions $L \times L$, $\mathbf{v} = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_L(t) \end{bmatrix}$, $\mathbf{r} = \begin{bmatrix} r_1(t) \\ r_2(t) \\ \vdots \\ r_L(t) \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} w_1(t) \\ w_2(t) \\ \vdots \\ w_L(t) \end{bmatrix}$ are vectors each of dimension L.

The network topology is assumed known (which G^0_{ji} are zero and which are not). As the network topology is problem data, it is embedded into the matrix description $\mathbf{G}$ in (4). An additional suppositions that the network is well-posed, or equivalently $\det(\mathbf{I} - \mathbf{G}) \neq 0$ must hold. It is also assumed that the network target module G^0_{ji} be identifiable [19–22] and the external excitation signals r_j, $j \in \mathcal{R}$ are uncorrelated to all noise signals $e_j \in \mathcal{E}_j$.

Finally, it is herein assumed that the target module G^0_{ji} is passive and this property should be preserved in model to be estimated.

Unlike existing methods in the literature, we present a novel approach to estimate a passive quadruple $(\mathbf{A}, \mathbf{b}, \mathbf{c}, c_0)$ of G^0_{ji} based on a time-domain experiment characterized by node measurements ($w_j(t)$, $w_k(t), k \in \mathcal{N}_j$, $r_k(t), k \in \mathcal{R}$), with $t = 1, \ldots, N$ and T_s denoted as the sampling time. The objective is thus to estimate a passive realization for a selected target module of a given dynamic network.

3. Discrete-time passivity

The concept of passivity derives from the general concept of dissipativity. Dissipative systems can be described via inequalities that account for energy/power balances. Formulating such inequalities entails defining functions whose variables describe the energy within the system as well as functions accounting for energy exchange between the system and its surroundings. Common formulations use a supply function $\Phi(\cdot)$ and a storage function $V(\cdot)$, the latter also known as a Lyapunov function. While the supply function describes energy exchange (input and output energy), the storage function describes the energy stored in the system. Eq. (5) gives an energy balance which can readily be interpreted as a physical system that cannot store more energy than the amount fed by the external source(s):

$$\dot{V}(x(t)) \leq \Phi(u(t), y(t)). \tag{5}$$

Passive systems are a special class within the broader class of dissipative systems [17]. For this special class, a LTI system, represented by a rational transfer function (or by Eq. (3)), is equivalently passive when satisfying the positive-realness bounded/positive-real lemma criteria [17,23]. As stated earlier, this property generalizes stability and causality [11,17].

The equivalence between passive behavior, positive/bounded-realness and positive/bounded-real lemma criteria are instrumental to build formulations that guarantee passivity for LTI systems, see [15,17,23].

The analysis called passivity assessment is the one that verifies whether a given system satisfies (or not) the passivity conditions. This assessment analysis can be made using various techniques including sweep methods for raw frequency-domain measured data [16]. A frequency-sweeping assessment test is based on the following condition (positive-realness):

$$G^*_{ji}(z) + G_{ji}(z) \geq 0, \tag{6}$$

for all z in which $z = e^{j\omega T_s}$, $\omega \in [0, 2\pi]$. G^*_{ji} denotes the conjugate transpose of G_{ji}. In the particular case of SISO systems, passivity requires the real part of $G_{ji}(e^{j\omega T_s})$ to be positive.

Alternatively, LMI-based assessments (positive-real lemma) require a parametric description such as a state-space realization to assess the passivity of a given system [11,17]. If there exists a symmetric and positive-definite ($\mathbf{P} = \mathbf{P}^T > 0$) such that:

$$\begin{bmatrix} \mathbf{A}^T\mathbf{P}\mathbf{A} - \mathbf{P} & \mathbf{A}^T\mathbf{P}\mathbf{b} - \mathbf{c}^T \\ (\mathbf{A}^T\mathbf{P}\mathbf{b} - \mathbf{c})^T & \mathbf{b}^T\mathbf{P}\mathbf{b} - 2c_0 \end{bmatrix} \leq 0, \tag{7}$$

with the system state-space matrices corresponding to a discrete-time realization, the system is said passive. In other words, passivity requires the LMI (7) to be feasible. Otherwise, the system is said to be non-passive. Feasibility of the LMI is both necessary and sufficient for satisfying system passivity [12].

Passivity enforcement is typically performed as a post-processing technique, aiming to convert a non-passive model into a passive counterpart. Two strategies exist for this purpose: perturbative and non-perturbative [17]. Perturbative methods achieve passivity by strategically modifying the state-space realization output parameter ($\mathbf{c}$) and feedthrough parameter (c_0), while keeping $\mathbf{A}$ and $\mathbf{b}$ unaltered. This enforcement is formulated as an optimization problem, often minimizing either the deviation from the original model or the magnitude of the applied perturbation [13]. Non-perturbative methods, in contrast, seek entirely new values for $\mathbf{c}$ and c_0 while maintaining the same $\mathbf{A}$ and $\mathbf{b}$. This is achieved by formulating a convex programming problem subject to specific constraints, such as Positive Real Lemma (PRL) written as Linear Matrix Inequality (LMI) conditions [8].

4. The frequency-domain approach

In what follows, we present a two-stage approach to identify passive local modules in a dynamic network. The main advantage of this approach lies in the fact that the method complexity does not depends on the network complexity as a whole but, rather on the complexity of the local environment, i.e., the number of incoming adjacent modules.

First, an indirect approach is used to compute the correlation between the excitation and node signals [2] aiming to reconstruct the node signals with information from the excitation signal. Then the nonparametric estimator, Local Polynomial Method (LPM) [24], is used to obtain a frequency response function (FRF) estimate for a selected target module. Its efficiency has already been proven to reduce leakage errors caused by the application of Fourier transform techniques to nonperiodic data [25]. It is particularly suitable for analyzing signals that are not periodic, as it can accurately estimate the FRF even from a finite data set. This combined procedure, that is, to reconstruct the node signals then to apply the LPM method, is known as indirect Local Polynomial Method (iLPM), as described in [18].

In the next stage, the estimated FRF is used as an input to the parametric estimator. As a result, this initial stage requires no specification of model order thus allowing the user select an order for the model subsequently.

This paper proposes a novel methodology for this second stage, termed here by Passive Vector Fitting (PVF). It constructs a sequence of passive models during the Sanathanan–Koerner iterations, ensuring that the final estimated model is inherently passive. This characteristic makes the method different from conventional VF, which relies on post-processing techniques applied to an estimated non-passive model.

This innovation brings the guarantee of passivity into the realm of local module identification for dynamic networks, addressing an open question in this field. The PVF algorithm estimates a passive parametric model (quadruple $(\mathbf{A}, \mathbf{b}, \mathbf{c}, c_0)$) for target local module G^0_{ji} from network data.

Let us start with a time-domain experiment characterized by node measurements $(w_j(t),\ w_k(t), k \in \mathcal{N}_j,\ r_k(t), k \in \mathcal{R})$, with $t = 1, \ldots, N$.

4.1. The nonparametric estimator as an indirect approach

We employ an indirect implementation of LPM. Traditionally, identifying a FRF for a given system via LPM involves selecting a set of signals as predictor inputs and then estimate a MISO model [24]. Conversely, the indirect approach hereby advocated entails the following steps [2]:

(1) Defining a MISO setup with w_j as output and w_k as inputs, $w_k \in \mathcal{N}_j$;
(2) Defining for each w_k a set of external excitation signals $\{r_m\}_k$, $m \in \mathcal{R}$ with a path from r_m to w_k;
(3) Estimating for each w_k the transfer functions between $\{r_m\}_k$ and the node w_k.
It consists of a decomposition of node signals w_k as $w_k = w_k^{\{r_m\}_k} + w_k^{\perp\{r_m\}_k}$ with $w_k^{\perp\{r_m\}_k}$ and r_m uncorrelated so that the component of w_k correlated with r_m be denoted $w_k^{\{r_m\}_k}$.
(4) Reconstructing the MISO setup of step (1) only with all correlated components $w_k^{\{r_m\}_k}$ to be used for the target module's FRF estimation.

In order to estimate a FRF using the iLPM, all components $w_k^{\{r_m\}_k}$ are first pre-processed by dividing it into overlapping segments of equal length. During this procedure, a narrow sliding processing window is employed. Within each segment, a polynomial of degree M is fitted to each $w_k^{\{r_m\}_k}$ using a least square approach. The polynomial coefficients are then used to compute the FRF of the MISO model from step (4) which includes the target module G_{ji}, using a Fast Fourier Transform (FFT). Further details on the implementation of iLPM, refer to [2,18,26].

The FRF of the target module obtained via the iLPM algorithm is denoted as $\widetilde{G}_{ji}(\omega_\kappa)$, where $\kappa = 0, \ldots, N_f$, N_f is the number of frequency samples.

4.2. Passive vector fitting

The objective is to compute a passive parametric model $\breve{G}_{ji}(\omega)$ so that $\breve{G}_{ji}(\omega) \approx \widetilde{G}_{ji}(\omega)$ for all frequencies $\omega = \omega_\kappa$ in a least square sense. To achieve this objective, we implement modifications to the Vector Fitting (VF) algorithm to include passivity conditions within the estimation process.

VF implementations are numerically robust implementations of the Sanathanan–Koerner/Steiglitz–McBride iterations [27,28] in which the minimization of the non-linear objective function is achieved iteratively via a sequence of linear least square problems. The modification includes passivity conditions as constraints in each iteration and is discussed as follows.

The target passive model $\breve{G}_{ji}(\omega)$ has the following structure:

$$\breve{G}_{ji}(z) = \frac{N(z)}{D(z)} = \frac{c_0 + \sum_{\alpha=1}^{n} \frac{c_\alpha}{z-p_\alpha}}{1 + \sum_{\alpha=1}^{n} \frac{\tilde{c}_\alpha}{z-p_\alpha}} \tag{8}$$

where $\breve{G}_{ji}(z)$ is an nth order transfer function parameterized by a set of poles $\{p_\alpha\}_{\alpha=1}^n$, $\{c_\alpha\}_{\alpha=0}^n$ and $\{\tilde{c}_\alpha\}_{\alpha=1}^n$. Also, we henceforth denote θ and $\tilde{\theta}$ the set of c_α and $\tilde{c}_\alpha$, respectively.

$\breve{G}_{ji}(\omega)$ admit a minimal state-space equivalence as in (3), namely $\{\mathbf{A}, \mathbf{b}, \mathbf{c}, d\}$ such that:

$$N(z) = \left((z\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{b} \quad 1\right)\theta$$
$$D(z) = 1 + \left((z\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{b}\right)\tilde{\theta}$$

The algorithm follows the subsequent steps.

4.2.1. Step 1

The algorithm initiates assigning an initial set of poles $\{p_\alpha\}_{\alpha=0}^n$. These poles are typically defined as lightly damped poles with imaginary parts logarithmically spaced along the frequency axis, as detailed in the previous section.

4.2.2. Step 2

Within each iteration, we seek to minimize the linearized version of the weighted least square error [27] defined as following:

$$\mathrm{J}(\theta,\tilde{\theta}) = \sum_{\kappa=1}^{N_f} W(z_\kappa)^2 \left| N(z_\kappa) - D(z_\kappa)\widetilde{G}_{ji}(z_\kappa)\right|^2, \tag{9}$$

where $W(z_\kappa)$ is a weighting function, defined as in [29], to consider measurements with resonance peaks and exhibit significant variations in magnitude. Then, the optimization problem which should be solved at each iteration is:

$$\begin{aligned} \min_{\theta,\tilde{\theta},\mathbf{P}} \quad & \mathrm{J}(\theta,\tilde{\theta}) \\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{A}^T\mathbf{P}\mathbf{A} - \mathbf{P} & \mathbf{A}^T\mathbf{P}\mathbf{b} - \mathbf{c}^T \\ (\mathbf{A}^T\mathbf{P}\mathbf{b} - \mathbf{c})^T & \mathbf{b}^T\mathbf{P}\mathbf{b} - 2c_0 \end{bmatrix} \preceq 0, \end{aligned} \tag{10}$$

This guarantees that a sequence of passive models is obtained during the VF iterations. Minimizing cost function (10) is equivalent to minimizing

$$\mathrm{J}(\theta,\tilde{\theta}) = \left\| \mathbf{M}\begin{bmatrix}\theta \\ \tilde{\theta}\end{bmatrix} - \mathbf{F} \right\|^2, \tag{11}$$

in which $\mathbf{F} = \begin{bmatrix}\Re(\mathbf{F}^c) \\ \Im(\mathbf{F}^c)\end{bmatrix}$ and $\mathbf{M} = \begin{bmatrix} \Re(\mathbf{M}^c) & \mathbf{1} & -\Re(\mathbf{M}^c) \\ \Im(\mathbf{M}^c) & \mathbf{1} & -\Im(\mathbf{M}^c) \end{bmatrix}$,

$$\mathbf{F}^c = \begin{pmatrix} \widetilde{G}_{ji}(z_1) \\ \widetilde{G}_{ji}(z_2) \\ \vdots \\ \widetilde{G}_{ji}(z_N) \end{pmatrix}, \mathbf{M}^c = \begin{pmatrix} W(z_1)(z_1\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{b} \\ W(z_2)(z_2\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{b} \\ \vdots \\ W(z_N)(z_N\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{b} \end{pmatrix},$$

superscript $(\cdot)^c$ stands for a complex valued matrix and both $\Re(\cdot)$ and $\Im(\cdot)$ stand for its real and imaginary parts, respectively. Eq. (11) is

$$\mathrm{J}(\theta,\tilde{\theta}) = \left[\mathbf{M}\begin{bmatrix}\theta \\ \tilde{\theta}\end{bmatrix} - \mathbf{F}\right]^T \left[\mathbf{M}\begin{bmatrix}\theta \\ \tilde{\theta}\end{bmatrix} - \mathbf{F}\right]. \tag{12}$$

Using the following QR decomposition $\mathbf{M} = \mathbf{QR}$ with $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$ and after some algebra we conclude that:

$$\mathrm{J}(\theta,\tilde{\theta}) = \left(\mathbf{R}\begin{bmatrix}\theta \\ \tilde{\theta}\end{bmatrix} - \mathbf{Q}^T\mathbf{F}\right)^T \left(\mathbf{R}\begin{bmatrix}\theta \\ \tilde{\theta}\end{bmatrix} - \mathbf{Q}^T\mathbf{F}\right) + \mathbf{F}^T\left(\mathbf{I} - \mathbf{Q}\mathbf{Q}^T\right)\mathbf{F}$$

or

$$J(\theta,\tilde{\theta}) = \mathbf{E}^T\mathbf{E} + \delta^2,$$

with $\mathbf{E} = \mathbf{R}\begin{bmatrix}\theta \\ \tilde{\theta}\end{bmatrix} - \mathbf{Q}^T\mathbf{F}$ and $\delta^2 = \mathbf{G}^T\left(\mathbf{I} - \mathbf{Q}\mathbf{Q}^T\right)\mathbf{G}$.

Minimizing $\mathrm{J}(\theta,\tilde{\theta})$ is equivalent to minimizing $\mathrm{J}(\theta,\tilde{\theta}) - \delta^2 = \mathrm{J}_\delta(\theta,\tilde{\theta}) = \mathbf{E}^T\mathbf{E}$. Using Schur complements leads to an epigraph convex formulation and a standard Semi-Definite Programming (SDP) problem results as in Eq. (13):

$$\begin{aligned} \min_{\mathbf{c},\tilde{\mathbf{c}},\mathbf{P},\mu} \quad & \mu \\ \text{s.t.} \quad & \begin{bmatrix} \mu & \mathbf{E}^T \\ \mathbf{E} & \mathbf{I} \end{bmatrix} \succeq 0 \\ & \mathbf{P} \succ 0 \\ & \mu \geq 0 \\ & \begin{bmatrix} \mathbf{A}^T\mathbf{P}\mathbf{A} - \mathbf{P} & \mathbf{A}^T\mathbf{P}\mathbf{b} - \mathbf{c}^T \\ (\mathbf{A}^T\mathbf{P}\mathbf{b} - \mathbf{c})^T & \mathbf{b}^T\mathbf{P}\mathbf{b} - 2c_0 \end{bmatrix} \preceq 0 \end{aligned} \tag{13}$$

with $\mathbf{A}$ and $\mathbf{b}$ problem data previously derived via the *Step 1* at first iteration or *Step 3* otherwise. This problem can be solved to find the optimal pair $[\mathbf{c}, \tilde{\mathbf{c}}]^T$ with the CVX solver [30].

4.2.3. Step 3

On terminating each iteration, poles are updated using:

$$\{p_1, \ldots, p_n\} = eig(\mathbf{A} - \mathbf{b}\tilde{\mathbf{c}}), \tag{14}$$

and them Step 2 is called.

The procedure is repeated recursively until convergence of (14).[1] On completion of the PVF algorithm, a passive model $\breve{G}_{ji}(z)$ is obtained as either a transfer function or an equivalent minimal state-space realization.

In summary, by adopting the approach hereby described accurate guaranteed passive models for the target modules are obtained. The two stages leading to a passive estimate are: (i) use of the iLPM to obtain a FRF for the local module(s) of interest; (ii) use of the PVF to achieve a passive parametric description based on the FRF by incorporating the PRL constraints into the optimization problem.

5. Numerical simulations

This section presents numerical simulations to demonstrate the practical application of our proposed methodology. We use a benchmark case study of a network system with complex interconnections, visualized in Fig. 1 and previously analyzed in [18]. All noise, external sources, and module specifications remain unchanged, except for module G_{31}^0, which is defined in this section. This specific modification serves a crucial purpose: to illustrate the estimation of a passive model for a passive system. This necessitates the original transfer function to be positive real. For the purpose of statistical analysis in this section, the model order of the system to be estimated, G_{31}^0, is assumed to be known.

The dynamics embedded in the network are defined as follows, with a sampling time of 0.01 s:

$$G_{32}^0 = \frac{0.09q^{-1}}{1+0.5q^{-1}};$$

$$G_{34}^0 = \frac{1.184q^{-1} - 0.647q^{-2} + 0.151q^{-3} - 0.082q^{-4}}{1 - 0.8q^{-1} + 0.279q^{-2} - 0.048q^{-3} + 0.01q^{-4}};$$

$$G_{14}^0 = G_{21}^0 = \frac{0.4q^{-1} - 0.5q^{-2}}{1+0.3q^{-1}}; H_1^0 = \frac{1}{1+0.2q^{-1}};$$

$$G_{12}^0 = G_{23}^0 = \frac{0.4q^{-1} + 0.5q^{-2}}{1+0.3q^{-1}}; H_2^0 = \frac{1}{1+0.3q^{-1}};$$

$$H_3^0 = \frac{1 - 0.505q^{-1} + 0.155q^{-2} - 0.01q^{-3}}{1 - 0.729q^{-1} + 0.236q^{-2} - 0.019q^{-3}}; H_4^0 = 1.$$

[1] For detailed insights into the convergence challenges associated with Vector Fitting, we recommend consulting Chapter 7 of the reference book [17].

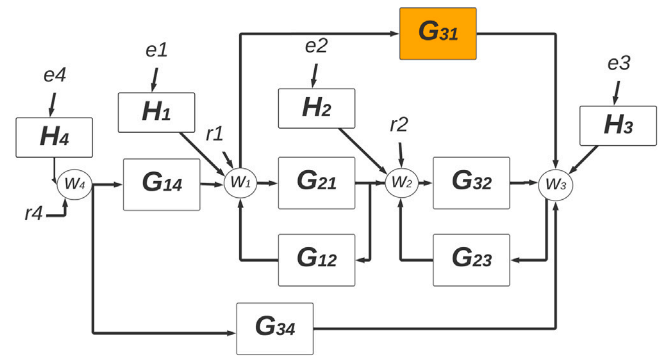

Fig. 1. Network example with 4 nodes, 3 reference signals and noise sources at each node [18].

5.1. A second-order passive model

$$G_{31}^{0}(z) = \frac{0.1717z^2 + 0.0202z - 0.1515}{z^2 - 1.771z + 0.8857}.$$

The MATLAB Toolbox for Dynamic Network Identification (beta version 0.1.0) [31] was employed to initially assess the identifiability of the target module G_{31}^0 based on the network's topology. The toolbox confirmed that G_{31}^0 is identifiable. As established in [32,33], for targets satisfying graphical identifiability conditions, ensuring an informative excitation signal suffices. We conducted 100 independent Monte-Carlo experiments in Simulink, each comprising 1300 samples. For each experiment, the data is generated using known reference signals $r_1(t)$, $r_2(t)$ and $r_4(t)$ that are realizations of independent white noise with variance of 0.1, also the noise sources $e_1(t)$, $e_2(t)$, $e_3(t)$ and $e_4(t)$ have variance 0.05, 0.08, 1, 0.1 respectively. Each experiment yielded a parametric state-space model for G_{31}^0.

To facilitate parameter comparison independent of units or scale, we employed the Coefficient of Variation (CV) defined as:

$$CV(x) = \frac{\delta_x}{\tilde{x}} \cdot 100$$

where x represents a normally distributed random variable, δ_x its standard deviation, and $\tilde{x}$ its mean.

We now discuss the parameterization choices for each stage of our approach.

iLPM: We proceed to estimate G_{31}^0 as a MISO identification problem with three inputs (w_1, w_2, w_4) and one output (w_3). For the iLPM, we opted for a second-degree polynomial. This choice improves the smoothness of the non-parametric estimate across the frequency range by providing broader bandwidth. However, it is important to be mindful of introducing bias through excessive bandwidth. Since the subsequent Vector Fitting step also introduces some smoothing, this initial parameterization becomes less critical.

As shown in [18], the minimum required number of frequencies for this configuration is 12. We opted for a wider bandwidth of 24 frequencies due to the presence of substantial noise. This reduces the impact of noise while minimizing bias error.

Passive Vector Fitting: During the PVF step, a linear weighting procedure is employed (see [29]) to estimate a passive second-order model. We have empirically concluded that a maximum of 50 iterations is enough to reach convergence.

The combined pole and residue estimates for G_{31}^0 across all experiments are summarized in Fig. 2. The analysis reveals $\mathbf{c} = [c_1 \; c_2] = [0.3243 \;\; -0.3036]$ suggesting dominant real and imaginary components of the poles at $a = 0.8855$ and $b = 0.3187$.

Fig. 3 showcases the frequency response curves for the data and the estimated model for the iLPM step and for the PVF step. In the

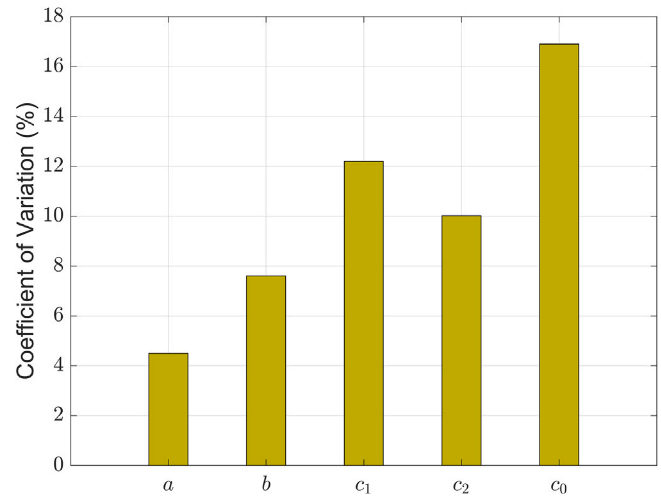

Fig. 2. Coefficient of Variation of the parameters of $\hat{G}_{31}$, estimated via 'iLPM+IVVF'.

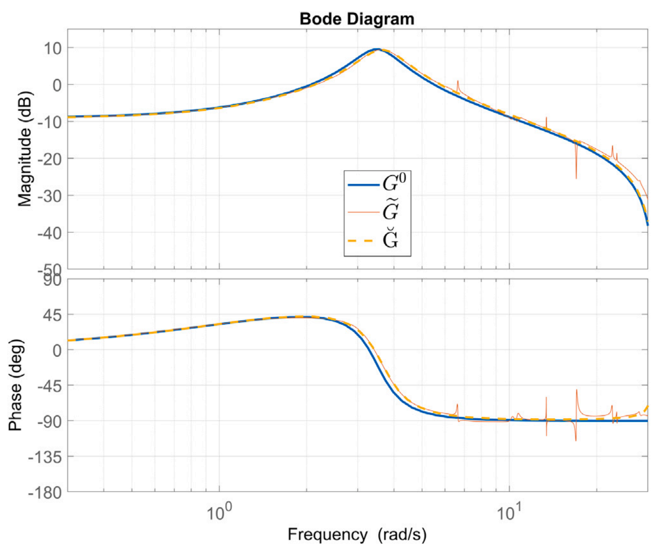

Fig. 3. Frequency response diagram of data and estimated models (2nd order).

particular instance depicted in Fig. 3, the RMSE between the $\breve{G}_{31}$ curve and the data is 0.07682. Across 100 experiments, the average error and variance between $\breve{G}_{31}$ and the data is 0.08634 and 0.0131, respectively. This figure effectively demonstrates the efficacy of the proposed methodology in achieving passivity while preserving the desired system dynamics.

This case study successfully demonstrates the effectiveness of incorporating energy consistency criteria into the developed approach for passive model estimation. By focusing on the system matrices $\mathbf{c}$ and c_0, our passive identification method ensures compliance with energy balance principles while minimizing parameter deviations. Importantly, the method proves successful in a complex 7-module network, highlighting its suitability for systems with intricate structures and correlated input and output signals.

5.2. A third-order passive model

Building upon the previous case study, we now consider a modified network scenario where the target module G_{31}^0 is defined as a 3rd order transfer function, with a sampling time of 0.01 s:

$$G_{31}^{0}(z) = \frac{10^{-4}(7.214z^3 - 7.018z^2 - 7.196z + 7.036)}{z^3 - 2.707z^2 + 2.418z - 0.7107}.$$

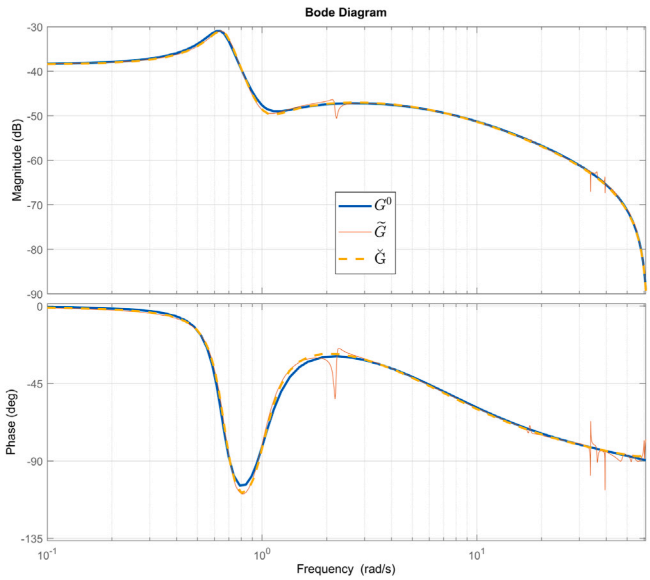

Fig. 4. Frequency response diagram of data and estimated models (3rd order).

All other network module specifications remain unchanged from the previous case study. The key objectives of this second case study are to demonstrate the ability of the Passive Vector Fitting (PVF) method to estimate a 3rd order passive model for the target module G^0_{31}, and to analyze the estimation accuracy and the required compensation to enforce passivity compared to the previous 2nd order case. Similar to the first case study, we conducted 100 independent Monte-Carlo experiments, each with 1300 samples of data generated using the same noise and reference signal specifications.

In the iLPM step, we increased the polynomial order to 3 to better capture the higher order dynamics of the target module. The minimum required number of frequency points was also increased to 18 to ensure sufficient bandwidth coverage.

During the PVF step, the algorithm was configured to estimate a 3rd order passive state-space realization for G^0_{31}. The maximum number of iterations was maintained at 50.

Estimation Accuracy: The average RMSE between the estimated 3rd order passive model $\breve{G}_{31}$ and the true 3rd order G^0_{31} across the 100 experiments was 0.001948, with a variance of 0.0009276. Fig. 4 illustrate the frequency response curves for the data and the estimated models and, for that instance, the RMSE is 0.002439.

Passivity Enforcement: The analysis revealed that 92% of the 3rd order model estimates required passivity enforcement, compared to 94% in the 2nd order case. The overall compensation remained moderate, suggesting that the proposed PVF method can effectively estimate passive models even for higher order target modules.

Fig. 5 provides a visual representation of the parameter variations in the estimated model.

6. Conclusions

This paper has demonstrated how an important gap in network identification can be effectively addressed with proven satisfactory results. The solution entails a sequential use of two well-established approaches iLPM (non-parametric) and VF (parametric) that combined form the semi-parametric algorithm herein developed. The latter parametric approach has been extended to include passivity constraints in its iterative procedure. Our findings indicate that incorporating passivity constraints into existing methodologies has had negligible impact on approximation metrics while ensuring consistent estimates with respect to energy constraints which is a mandatory property under complex interconnection of passive systems.

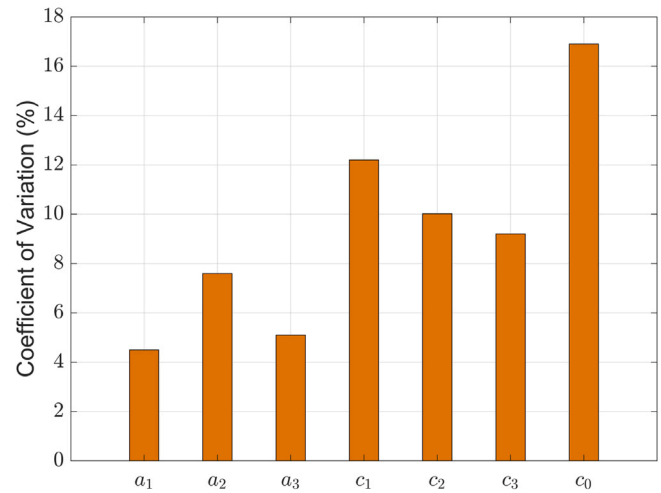

Fig. 5. Coefficient of Variation of the parameters of $\hat{G}_{31}$, estimated via 'iLPM+IVVF'.

CRediT authorship contribution statement

Lucas F.M. Rodrigues: Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Gustavo H.C. Oliveira:** Supervision, Conceptualization. **Lucas P.R.K. Ihlenfeld:** Supervision. **Ricardo Schumacher:** Methodology. **Paul M.J. Van den Hof:** Supervision, Software.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Lucas Farias Maciel Rodrigues reports financial support was provided by Federal University of Parana. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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