

Integrated Dynamic Optimization and Control in Reservoir Engineering using Locally Identified Linear Models

Gijs van Essen, Amin Rezapour, Paul M.J. Van den Hof and Jan Dirk Jansen

Abstract—Studies on dynamic real-time optimization (D-RTO) of waterflooding strategies in petroleum reservoirs have demonstrated that there exists a large potential to improve economic performance in oil recovery. Unfortunately, the used large-scale, nonlinear, physics-based reservoir models suffer from vast parametric uncertainty and generally poor short-term predictions. This seriously limits the industrial and economic feasibility of a production strategy based at long-term economic objectives alone. In this work, a two level optimization and control strategy is investigated, consisting of a D-RTO and model predictive control (MPC) level. In this structure, long-term economic performance is addressed by the design of optimal reference trajectories, while reliability of reaching these long-term goals is managed through a short-term tracking control problem, based on locally identified linear models.

I. INTRODUCTION

The production of oil from petroleum reservoirs is a typical example of a large-scale dynamical system where complex physics governs the nonlinear dynamics of the underlying models. Operations are classically driven by decisions based on operator experience and supported by scenario studies. Recent improvements in dynamic reservoir modeling as well as the introduction of more enhanced subsurface measurement and control devices have created a foundation for field-wide control and optimization of oil production [1], [2], [3].

Oil is produced from subsurface reservoirs. In these reservoirs the oil is contained in the interconnected pores of the reservoir rock under high pressure and temperature. The depletion process of a reservoir generally consists of two production stages. In the primary production stage the reservoir pressure is the driving mechanism for the production. In the secondary production stage liquid (or gas) is injected into the reservoir using injection wells. The most common secondary recovery mechanism involves the injection of water and is referred to as waterflooding. It serves two purposes: sustaining reservoir pressure and sweeping the oil out of pores of the reservoir rock and replacing it by water.

This research was carried out within the context of the Integrated Systems Approach to Petroleum Production (ISAPP) knowledge centre. ISAPP is a joint project between Delft University of Technology (TUD), Shell International Exploration and Production (SIEP), and the Dutch Organization for Applied Scientific Research (TNO).

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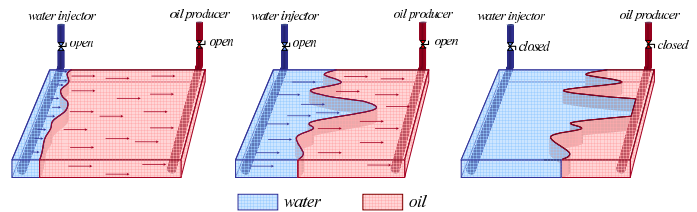


Fig. 1. Schematic representation of waterflooding at different moments in time using a (horizontal) injection and production well. Due to heterogeneity of the rock, water does not move uniformly towards the production well.

In Figure 1 a (schematic) representation of the waterflooding process is presented.

The waterflooding process is nonlinear, due to the different fluid properties of oil and water [4]. This nonlinear behavior strongly influences how the oil-water front moves through the reservoir. When the oil-water front reaches a certain well, it start producing water (water-breakthrough). The time between start of water injection and water-breakthrough may take several years. A reactive flooding strategy, based on measuring the oil-water ratio in each well and applying a shut-in threshold, is most frequently used in practice. However, such a reactive strategy often gives poor 'sweep efficiency', i.e. in certain regions the oil is not properly drained because the water passes the oil by, as can be observed in Figure 1.

Moving from a reactive production strategy to a more proactive one is difficult. First of all, feedback control is not feasible due to the long time constants of the process from start injection to water-breakthrough. Secondly, the process is essentially a (unique) batch process, ruling out any learning control. Therefore, in most oil companies, physics-based reservoir models are used, of which an example is shown

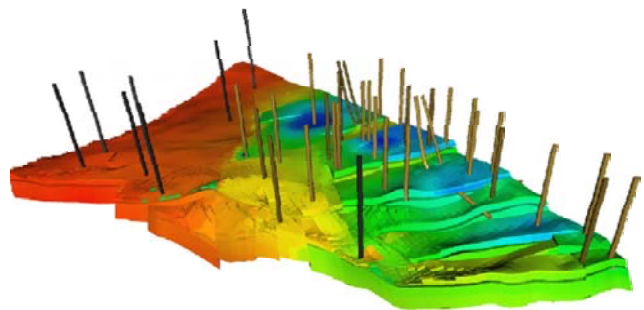


Fig. 2. Example of a large-scale, physics-based, numerical reservoir model with a large number of wells.

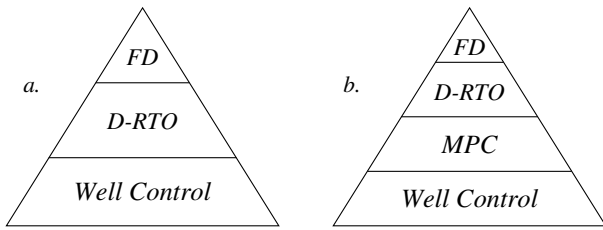


Fig. 3. Management structure of an oil field consisting of different control layers. In [2] control of the wells is directly manipulated by the D-RTO layer resulting in 3 control layers (a). In this work, an additional MPC layer is introduced between D-RTO and well control (b) as is common in continuous process control.

in Figure 2, that provide long-term predictions of the fluid flows through a reservoir. Based on these models, a proactive (open-loop) control strategy can be designed.

Introduction of a dynamic real-time optimization (D-RTO) into reservoir management using such a numerical reservoir model, can potentially increase (economic) performance by more than 15% [5], [6], [7]. This approach corresponds to the field management control structure, as is depicted in Figure 3.a, where the bottom layer deals with the basic control (stability) of wells, operational decisions of the wells are prescribed by the results of the D-RTO layer and at the top a field development (FD) layer determines the design of the field in terms of the number, location and type of wells. However, when an optimized production strategy is (open-loop) applied to an oil reservoir the expected increase in economic performance is generally not realized, since reservoir models usually suffer from vast uncertainty in their predictions.

In [8] this problem is addressed through a robust optimization (RO) approach which uses an ensemble of reservoir models that reflects modeling uncertainty. In [2], a procedure is introduced that involves a sequential execution of parameter and state estimation, and dynamic real-time optimization (D-RTO). However, in this structure, feedback is only implicitly present in through (possible) state estimation at certain moments in time. In between these moments control is applied in open-loop.

In this work, we investigate the value of introducing feedback into the control structure through the use of locally identified, linear models. These models are assumed to have better short-term prediction accuracy than the physics-based reservoir models, because the model structure of the large-scale models is aimed at capturing the slow, reservoir-wide dynamics. Also, adapting the parameter estimation problem such that the quality of short-term predictions is improved (at the expense of long-term accuracy) is difficult: they may still be dominated by the prior information incorporated in the reservoir model to improve long-term predictions. A predictive control (MPC) layer is introduced in the field management structure as shown in Figure 3.b, which is analogous to the current industrial state-of-the-art approach for process operation [9], [10]. Based on a large-scale, physics-based nonlinear reservoir model long-term reference trajectories are defined through D-RTO using an economic

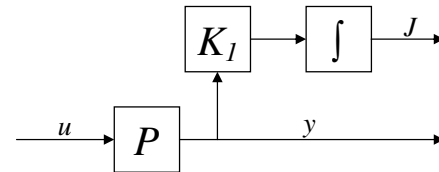


Fig. 4. Block diagram of an open-loop waterflooding procedure, where J represents an economic performance measure.

cost function [7], [11], [12]. Tracking of these reference trajectories is subsequently done through MPC, using the locally identified linear models as prediction model. It should be noted that since the optimal reference trajectories are based on an uncertain model, they will be sub-optimal for the real system.

The approach as described in the previous paragraph is closely related to the two-level optimization and control strategy as described by [13], [14] and [15]. However, a distinction can be made in the purpose of the low-order models in the MPC loop. Instead of serving as a locally valid, short-term proxy, the low-order models are a supplement to the large-scale model. As a result, consistency of the models is not guaranteed, nor required.

II. RESERVOIR MODELING USING LOCAL LINEAR MODELS

Figure 4 shows a block diagram of an open-loop waterflooding procedure, where P represents the oil reservoir. The controls u represent the flow rates of the water injection wells q_i and bottom-hole pressures of production wells p_{bh} . The N_y outputs - denoted by y - involve the production flow rates of both oil (q_o) and water (q_w). Block K_1 is a linear mapping of y in \mathbb{R}^{N_y} to economic revenues in \mathbb{R}^1 and contains oil price and water production costs. As a result, J represents cumulative economic revenues, where economic life-cycle performance of a reservoir is evaluated at a specific time T , relating to the end of the life of the reservoir.

Reservoir P prescribes the dynamic relationship between inputs q_i and p_{bh} and outputs q_o and q_w . Ideally, an identified model \hat{P} would - on the basis of input/output data - describe the same input/output mapping. However, the ratio between q_o and q_w (watercut) depends strongly non-linear on the inputs. Besides this, past data does not contain information about this non-linear relationship. To that end, combined oil and water flow rates q_i (liquid rates) are used as outputs in the identification of a data-driven linear model of the reservoir. In Figure 5, this is represented through block K_2 , which involves a linear mapping from dimension \mathbb{R}^{N_y} to $\mathbb{R}^{\frac{1}{2}N_y}$. It should be noted, that due to K_2 , perfect tracking of a reference r does not guarantee a zero error between the real and predicted outputs y and \hat{y} , which may consequently lead to a poorer economic performance than expected. Besides this, K_2 causes the number of outputs N_y to always be smaller than the number of inputs N_u .

To capture all relevant dynamics in an identified model,

input u must be persistently exciting during the length of the experiment. Design of such an experiment can be done on the basis of a physics-based large-scale model of the reservoir, if available. This approach was also adopted in the simulation study presented in this paper. System identification of MIMO systems can be carried out through several methods, e.g. prediction error identification (PEI) or subspace identification (SubID). In this work, SubID was chosen to identify a prediction model for the model predictive controller.

III. MODEL PREDICTIVE CONTROL IN WATERFLOODING

The model predictive controller, as presented in Figure 5, acts as a tracking controller for the reference trajectories, denoted by r . In this application, r corresponds to desired liquid flow rates of the production wells.

A cost minimizing control u is calculated over a relatively short time horizon, using an observer to estimate the initial conditions of the identified model. It should be noted that due to $N_y < N_u$, the minimization problem may become ill-posed. Only part of the control input will be implemented after which the process from state estimation to application of u is repeated. For the tracking problem of liquid production from reservoirs, the (discrete) quadratic cost function V that is evaluated over the receding prediction horizon is expressed by:

$$V(u) = \sum_{k=1}^N (\hat{q}_{l,k+1} - r_{k+1})^T R (\hat{q}_{l,k+1} - r_{k+1}) + (u_k - \bar{u}_k)^T Q (u_k - \bar{u}_k) \quad (1)$$

where k is the time step index, N the number of time steps of the prediction horizon, \hat{q}_l are the predicted liquid production rates, r the desired (reference) liquid rates, u the control input and \bar{u}_k a preferred control input. Matrices $Q \in \mathbb{R}^{N_u \times N_u}$ and $R \in \mathbb{R}^{N_y \times N_y}$ are positive semi-definite weighting matrices. The error between u_k and \bar{u}_k is incorporated in the objective function to find the minimizing tracking error control closest to preferred input \bar{u}_k , thus realizing a unique solution.

As the system moves away from the working point around which the LTI model was identified, its prediction accuracy will decrease and re-identification is required. In doing so, the benefit of improved prediction must be evaluated against

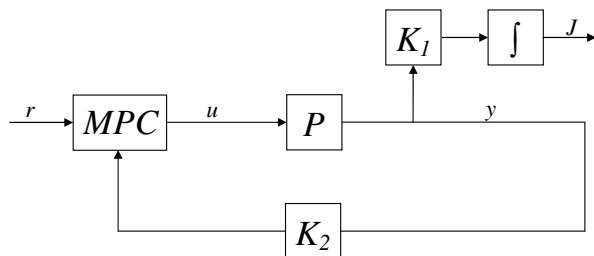


Fig. 5. Block diagram of an closed-loop waterflooding approach, using MPC for tracking control of the liquid flow rates of each production well.

the drop in tracking performance during the experiment. However, how to determine the best instant to initiate another identification experiment is not explored any further in this work, but will be the subject of future research.

IV. SIMULATION STUDY

Evaluation of the MPC scheme described in the previous section is restricted to numerical experiments, because of the long time constants and life-cycle of reservoirs, the inability to repeat the process for evaluation, and the high stakes involved in oil production. To that end, a numerical model must be used to represent reality. A detailed description of the underlying equations of such a numerical model can be found in [4].

In the example presented here, an oil reservoir is considered with 8 water injection and 4 production wells, as shown in Figure 6. Its geological structure involves a network of fossilized meandering channels in which the flowing fluids experience less resistance. The life-cycle of the reservoir covers a period of 11.5 years. Of this reservoir a large-scale model P was created to serve as an (unknown) real reservoir 'behind the curtain'. A validation of method against an (unknown) synthetic 'truth' is often used in reservoir engineering because of the very long time constants of the process and the economic stakes involved [16]. In order to let P provide more realistic predictions of short-term dynamic behavior of the reservoir, a very fine spatial discretization around the wells was adopted and a (relatively) very short time step size was chosen of 0.25 days. However, no artificial measurement noise, nor disturbances were applied to P or its (synthetic) input/output data.

A second reservoir model M was created of the oil reservoir, which serves as available (known) model to provide long-term predictions, to be used for D-RTO and design of an identification experiment. No grid refinement around the wells was used and a time step size of 30 days was adopted. This (coarse) spatial and time discretization is of the same order as most reservoir models used by oil companies. Besides the more coarse discretization, reservoir model M deviates from P in its geological structure, which has its channels in a different flow direction, as can be observed in Figure 7

A. Economic Life-Cycle Optimization

In the control structure presented in Section I, reference trajectories r are determined by the D-RTO layer. In reference to the waterflooding procedure presented in [2] an economic objective function was adopted in terms of net present value (NPV), which is defined in discrete form as:

$$J = \sum_{k=1}^{k_T} \left[\frac{r_o \cdot q_{o,k} - r_w \cdot q_{w,k}}{(1+b)^{\frac{k}{T}}} \cdot \Delta t_k \right], \quad (2)$$

where r_o is the oil price, equal to $56.6 \frac{\$}{m^3}$ and r_w the water production costs, equal to $6.3 \frac{\$}{m^3}$, which are both assumed

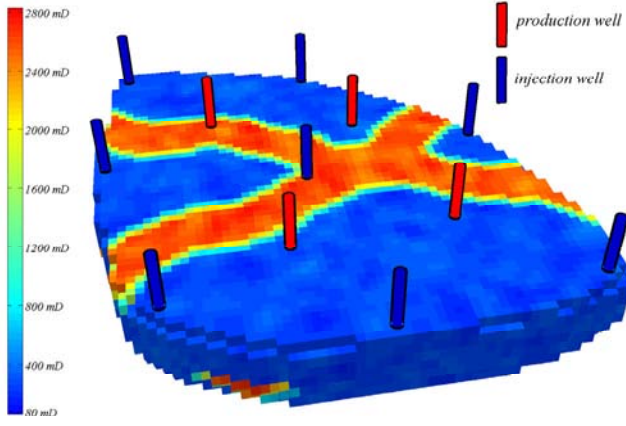


Fig. 6. Oil reservoir containing 8 injection wells and 4 production wells. Channels in the reservoir have larger permeability (in Darcy), which is a measure inversely proportional to the flowing resistance.

constant. k_T is the time step index relating to final time T and Δt_k the time interval of time step k in [day]. The term b represents the discount rate for a certain reference time τ_r . This term is added to take into account the time value of money. In (2), $b = 0.1$ and $\tau_r = 365.25$ days.

The reference trajectories r are calculated based on (known) reservoir model M . Using M , the following optimization problem was solved:

$$u^* = \arg \max_u J_{k_T} \quad (3)$$

$$s.t. \quad f(x_{k+1}, x_k, u_k) = 0, \quad x_0 = \hat{x}_0, \quad k = 1, \dots, k_T \quad (4)$$

$$g(u_k) \leq 0 \quad (5)$$

where f represents the model equations of reservoir model M , \hat{x}_0 the initial conditions and g the constraints that act on the inputs. These constraints involve a lower bound on q_i and p_{bh} of $0 \frac{m^3}{day}$ and 375 bar respectively, and an upper bound on q_i of $1,590 \frac{m^3}{day}$.

A number of methods are available for dynamic optimization on large scale problems [17], [18], [19]. Although the capacity of *simultaneous* methods to handle large-scale problem has increased considerably over the recent years, models of order 10^6 are still difficult to solve in this manner. Although *sequential* methods generally require repeated numerical integration of the model equations, only the control vector is parameterized and as a result can deal with larger problems. Therefore, a single shooting method was adopted in this work to solve equations (3)-(5) using a system of adjoint equations to efficiently calculate the gradients, as described in [7]. Note that due to the absence of path constraints, preserving feasibility of the solution is relatively easy. Solving dynamic optimization problem (3)-(5) leads to an expected maximum NPV of 596 M\$. The reference trajectories r used for MPC control are determined through:

$$r_k = K_2 \cdot \hat{y}(u^*)_k, \quad k = 1, \dots, k_T. \quad (6)$$

where \hat{y} represents the outputs (oil and water flow rates) predicted by M .

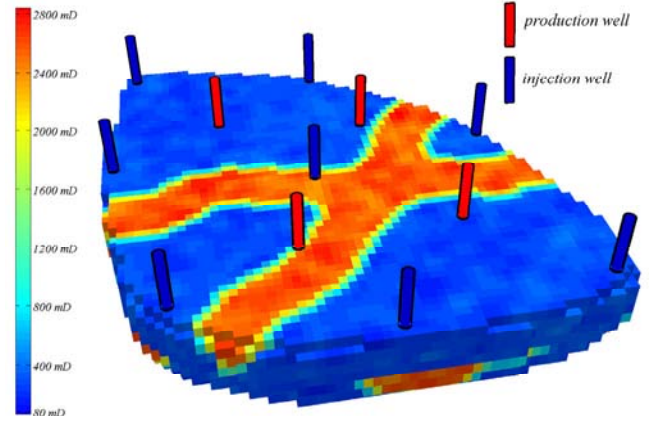


Fig. 7. Oil reservoir model containing 8 injection wells and 4 production wells. The model differs from the model shown in Figure 6 in the direction of the channels, the absence of grid refinement and the larger time step size of 30 days.

B. System Identification

As described in Section II, the inputs involve the water injection rates q_i of the 8 injection wells and the bottom-hole pressures p_{bh} of the 4 production wells. The reference trajectories r involve the liquid rates q_i of the 4 production wells. As such, an eight inputs/four outputs model needs to be determined.

The waterflooding process is non-linear and as a result the prediction accuracy of a linear model will decrease when the prediction horizon increases. However, in this first experiment to introduce MPC in Reservoir Engineering, a linear model is only identified at the start of production for simplicity reasons. The duration of the experiment was chosen using the rule of thumb of minimal 5 times the largest time constant. Through step response analysis on reservoir model M , the largest time constant was estimated and the minimal duration of the experiment was estimated at 75 days.

For q_i and p_{bh} , RBS excitation signals were generated. From step responses on M for each input, it was found that the response to changes in q_i were much slower than responses to changes of p_{bh} . In order to amplify the low frequency content of excitation signal of the injection rates, the clock period of the RBS signal was set to 3 sample times (of 0.25 days). The amplitudes the RBS signals were set at $1590 \frac{m^3}{day}$ and 1 bar for q_i and p_{bh} respectively, using M to determine that the effect on the outputs was significant and proportional.

In order to maintain good economic performance, the RBS signals with zero mean were superimposed upon the preferred inputs \bar{u}_k which are taken equal to u^* . However, whenever addition of the RBS signal led to infeasibility with respect to (5), the mean value was moved up or down until feasibility was reached. In the identification experiment, the first 25 days of data were omitted after which all initialization effects had died out. Note that, due to the superposition of the RBS signal on u^* , the spectrum of the excitation signal of the producers is no longer flat, but has a larger weight in the low frequency range. The sampling frequency of the

measured outputs y is determined by the time step size of P , which in this example was equal to 0.25 days.

The identification experiment was conducted in open-loop, since the depletion process is inherently stable. Based on these data, an 8th-order subspace identification (SubID) model \hat{P} was identified. Subspace identification was used because of its simple structure that is well suited for MIMO systems, and more importantly that it is computationally efficient. Figure 9 shows the simulation fit of the model with respect to the measured output data for all four producers. It can be observed that the output of the identified model has satisfying accuracy.

C. Model Predictive Control

The MPC control involves minimization of a cost function as described by (1). The desired input \bar{u} is chosen equal to u^* as a best guess to maximize economic life-cycle performance. Weighting matrix R was taken equal to unity. Weighting matrix Q was used to weigh only deviations of the injection rates q_i from optimal inputs u^* , such that tracking is mainly realized through changes in the bottom-hole pressures p_{bh} of the production wells. The nonzero elements of Q were chosen equal to unity. In this experiment, prediction horizon N was chosen equal to 28, which is approximately equal to the rise time of the slowest step response of M . Each time step, the minimization problem is solved for the (moving) prediction horizon, which involves two sequential steps: State estimation and Quadratic Programming (QP).

1) *State Estimation*: In this simulation study, no artificial noise was added to the measurements. As a result, the state estimation problem can be attacked quite straightforwardly using a Luenberger observer. The observer gain was chosen such that the poles of the observer were converge approx-

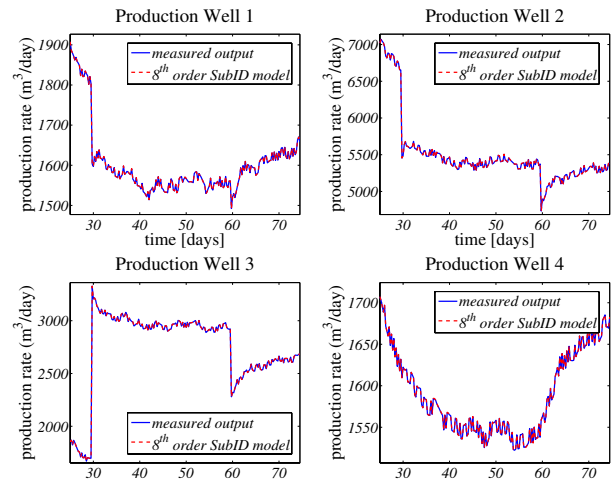


Fig. 9. Simulation fit of the 8th-order SubID model with respect to the measured output data.

imately 10 times faster than the poles of the system. Note however that alternative choices for state estimation may be considered, e.g. Kalman filtering.

2) *Quadratic Programming*: To solve the minimization of (1) subject to the inequality constraints on the inputs (5), a QP problem needs to be solved. In this QP problem, the state estimation serves as initial condition for the predictions of \hat{P} . In this experiment, a custom made QP solver is used, implemented in the used reservoir simulator.

The control of each well - determined by the MPC controller - is applied and re-calculated at every time step of 0.25 days of the 'real' reservoir P and can be observed in Figure 8.

D. Results

The results of adding a MPC layer to control production of P are compared to direct, open-loop application of optimal inputs u^* to P . Performance is both evaluated in terms of tracking performance and NPV. Tracking performance can be observed in Figure 10. In Table I, the NPV's of the open-loop application of u^* on P and the MPC controlled reservoir are shown and compared to the expected maximum NPV determined by the D-RTO layer. Figure 10 shows the reference and output trajectories for both the open-loop and MPC controlled case for each of the four production wells over the life of the field. In each of the four plots, 4 different stages can be identified using MPC. In the first 75 days of production, the identification experiment is conducted where the optimal inputs u^* serve as mean values. During this period the error is large because of the

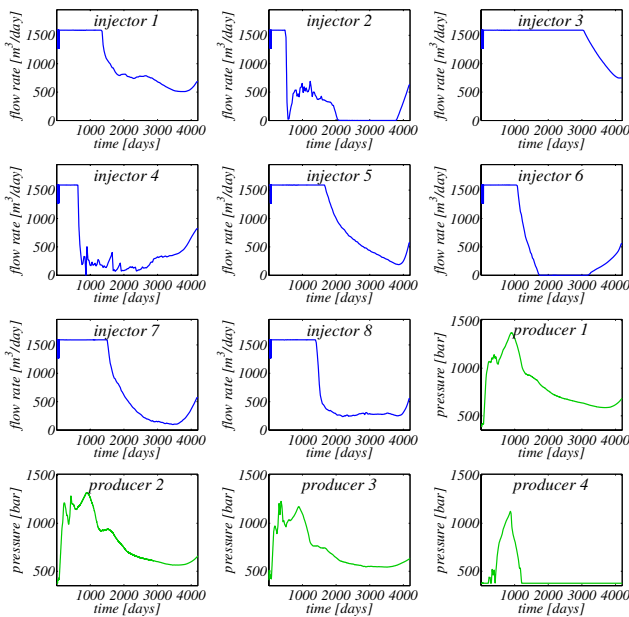


Fig. 8. Control of each well as determined by the MPC controller. It is applied and re-calculated at every time step of 0.25 days of the 'real' reservoir P .

TABLE I
ECONOMIC PERFORMANCE

	NPV	% change
Maximum predicted by M	596 M\$	-
Open-loop application u^* to P	558 M\$	-6.4 %
MPC control of P tracking r	594 M\$	-0.5 %

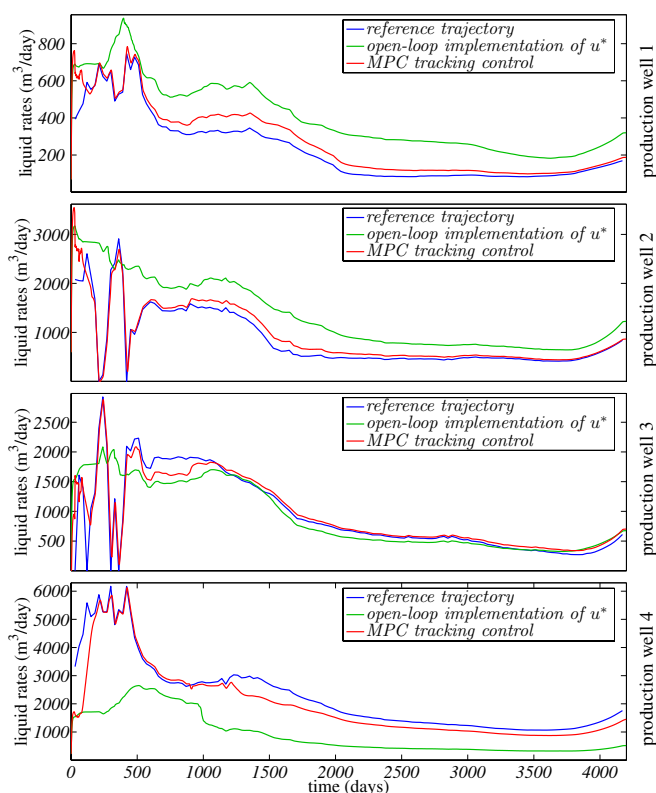


Fig. 10. Reference and output trajectories for both the open-loop application and MPC controlled case for each of the four production wells over the life of the field.

model error between the reservoir model M and 'reality' P , while the MPC controller is not active yet. From 75 days to approximately 500 days tracking performance is good due to activation of the MPC controller. After 500 days tracking performance decreases, however still outperforming open-loop control. This drop is the result of water breakthrough in the production wells, which has a strong nonlinear effect of the dynamics. After approximately 3000 days tracking improves, due to the fact that mainly water is produced resulting in a more linear response to the inputs.

V. CONCLUSION

The introduction of an output tracking controller in oil production is aimed at attenuation of uncertainty effects on the predictions. As a result, the system is assumed to give better economic performance than open-loop control. This approach is supported by the work presented in [14]. Although as of yet only the groundwork for MPC tracking control in oil production has been laid out in this paper, from the results of the numerical example the following observations can be made:

- It is possible to obtain a predictive reservoir model through system identification, which gives accurate predictions for a time horizon that is relatively short, but long enough to realize MPC receding horizon control.
- The results of reference tracking through MPC are promising. However, even better results are expected when frequent re-identification of the local linear model

is conducted over the length of the field's life-cycle.

It should be noted that the presented integrated D-RTO and MPC approach can quite easily be combined with alternative methods that are aimed at counteracting the negative effects of uncertainty on economic performance, such as the robust optimization procedure presented in [8] and the sequential D-RTO and data-assimilation approach presented in [2].

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