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An iterative scheme for identification and control design based on coprime factorizations

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Abstract

The topic of this paper is the design of a high performance compensator for an imprecisely known plant by means of approximate identification and model-based control design. For this purpose identification and control design have to be treated as the joint problem of finding a nominal model whose compensator achieves a high performance for the plant. This joint problem is tackled by an iterative scheme of repeated identification and control design. The scheme combines a frequency-domain identification technique and a robust control design method that are conceived in terms of coprime factors. An example attests to the utility of our iterative scheme for high performance control design.

1 Introduction

Recent years have seen a growing interest in the use of system identification as a means to build models for control design. The traditional identification methods deliver a model in the form of a linear time-invariant finite dimensional system. Such a model, called a nominal model, approximately describes the dynamics of the plant of concern, so that a robust controller is needed.

Many robust control design methods demand a quantification of the so-called 'model-error'. This has motivated the development of identification techniques that estimate an upper bound on the deviations between a plant and its nominal model [5, 8, 10, 4]. With this upper bound a controller can ideally be designed such that some performance is guaranteed for the plant. However, by itself, a tight estimated upper bound is not sufficient for high performance control design. As the bound cannot be smaller than the *true* model-error, it is the true model-error that limits the achievable performance. Hence a well-suited nominal model is needed.

In this paper we address the identification of a nominal model for high performance control design. The controller $C_{\hat{P}}$ designed from $\hat{P}$ has to achieve a high performance for the modelled plant P and a similar performance for the nominal model $\hat{P}$. The former is the true control objective; the latter is needed in order that we are confident about the compensator $C_{\hat{P}}$. Simultaneous high performances are accomplished, if the feedback system composed of the nominal model $\hat{P}$ and its own high performance compensator $C_{\hat{P}}$ approximately describes the feedback system containing the plant P and the same compensator $C_{\hat{P}}$. On the other hand if there does not exist a compensator that achieves a high performance for the plant as well as for the nominal model, then the nominal model is not suited for this control design problem.

The quality of each candidate nominal model depends on its own compensator and vice versa. Hence the problem of designing a high performance compensator for an imprecisely known plant boils down to a *joint problem* of approximate identification and model-based control design. This joint problem can be solved by means of system identification and model-based control design, only if these procedures are embedded in an iterative scheme [15]. We elaborate an iterative scheme, in which each identification is based on new data collected while the plant is controlled by the latest compensator. Each new nominal model is used to design an improved compensator, which replaces the old compensator. This iteration is closely related to adaptive control. However, in the proposed iteration we can study identification and control design separately, whereas these procedures are completely intertwined in adaptive control.

A few iterative schemes proposed in literature have been based on the prediction error identification method together with LQG/LTR control design [1], with LQG control design [6] and with LQ control design [18]. Alternatively, in [9] the identification and control design are based on covariance data. In [12] an iteration is used to build prefilters for a control-relevant prediction error identification from one open-loop dataset.

Our iterative scheme is composed of a robust control design method and a frequency domain identification technique that are conceived in terms of coprime factorizations. In Section 2 the control design method is delineated and the control objective is used to link identification and control design together. The resulting problem of control-oriented approximate identification from closed-loop data is tackled in Section 3. For this purpose we represent the imprecisely known plant by a coprime factorization that is based on a dual generalized Youla parameterization. The identified nominal model is used for an enhancement of the controller as discussed in Section 4. The proposed iteration is applied in a simulation study in Section 5, and the final section contains some concluding remarks.

2 A link between identification and control design

We adopt the following control design paradigm from [2]. The feedback configuration of interest is the interconnection $H(\hat{P}, C)$, which is depicted in Fig. 1. The transfer matrix $T(\hat{P}, C)$ defined as

$$T(\hat{P}, C) \doteq \begin{bmatrix} \hat{P}(I+C\hat{P})^{-1}C & \hat{P}(I+C\hat{P})^{-1} \\ (I+C\hat{P})^{-1}C & (I+C\hat{P})^{-1} \end{bmatrix} \quad (1)$$

maps $\mathrm{col}(r_2, r_1)$ into $\mathrm{col}(\hat{y}, \hat{u})$. This transfer matrix is called the nominal *feedback matrix*, because it embodies all feedback properties like disturbance and noise attenuation, sensitivity, stability and robustness margins. The model-based controller $C_{\hat{P}}$ is derived from the nominal model $\hat{P}$ according to

$$C_{\hat{P}} = \arg\min_{C} \|T(\hat{P}, C)\|_\infty. \quad (2)$$

The resulting controller is optimally robust against stable perturbations of the normalized right coprime factors of $\hat{P}$ (see [2, 17] for details). At the same time this controller $C_{\hat{P}}$ pursues some traditional control objectives like a small sensitivity at the lower frequencies and a small complementary sensitivity at the higher frequencies [11].

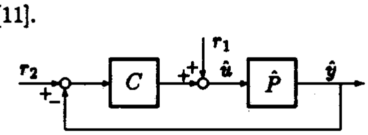

Fig. 1: Feedback configuration for control design

Conformably to (2) the nominal performance is high, if $\|T(\hat{P}, C_{\hat{P}})\|_\infty$ is small. We examine the performance norm of

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the actual plant P by the upper bound

$$\|T(P,C_{\hat{P}})\|_\infty \le \|T(\hat{P},C_{\hat{P}})\|_\infty + \|T(P,C_{\hat{P}}) - T(\hat{P},C_{\hat{P}})\|_\infty. \quad (3)$$

The left term reflects the performance of the controlled plant. The nominal performance norm $\|T(\hat{P},C_{\hat{P}})\|_\infty$ is minimized by the design of (2); and $\|T(P,C_{\hat{P}})-T(\hat{P},C_{\hat{P}})\|_\infty$ is the 'worst-case' performance degradation due to the fact that $C_{\hat{P}}$ has been designed for the nominal model $\hat{P}$ rather than for the plant P.

With the above inequality we can make more precise the implications of the high performance control design problem. The model-based compensator $C_{\hat{P}}$ has to achieve a high performance for the nominal model $\hat{P}$, and thus $\|T(\hat{P},C_{\hat{P}})\|_\infty$ must be small. The same compensator $C_{\hat{P}}$ has to achieve a similar performance for the plant P. Therefore we require that the performance degradation $\|T(P,C_{\hat{P}})-T(\hat{P},C_{\hat{P}})\|_\infty$ is much smaller than the nominal performance norm $\|T(\hat{P},C_{\hat{P}})\|_\infty$. If the latter is accomplished, then the nominal feedback matrix $T(\hat{P},C_{\hat{P}})$ approximately describes the feedback properties of the controlled plant, i.e. of $H(P,C_{\hat{P}})$.

As the control design of (2) pursues a small nominal performance norm $\|T(\hat{P},C_{\hat{P}})\|_\infty$, the remaining task for the approximate identification would be to find such a nominal model $\hat{P}$ that the performance degradation $\|T(P,C_{\hat{P}})-T(\hat{P},C_{\hat{P}})\|_\infty$ is relatively small. This approximate identification problem cannot be solved straightforwardly, because the compensator $C_{\hat{P}}$ is not available prior to the identification. This explains once more that the problems of approximate identification and model-based control design have to be treated as a joint problem.

Remark 2.1 The upper bound of (3) is used to express the identification objective in terms of the control objective of (2). The same approach applies to any other control design method that optimizes a norm or a distance function of the nominal feedback matrix $T(\hat{P},C_{\hat{P}})$. As explained in [14] these methods include LQ control design and the H_∞-optimization of a weighted sensitivity. □

We propose the following iterative scheme to tackle the joint problem of approximate identification and model-based control design. In the i-th step we obtain data from the plant, while it operates under feedback by C_{i-1}. The nominal model $\hat{P}_i$ is identified according to

$$\hat{P}_i = \arg\min_{\bar{P}\in\mathcal{P}(\theta)} \|T(P,C_{i-1}) - T(\bar{P},C_{i-1})\|_\infty \quad (4)$$

where $\mathcal{P}(\theta)$ is the set of parameterized candidate models. The resulting $\hat{P}_i$ involves a small performance degradation for C_{i-1}, so that this nominal model $\hat{P}_i$ approximately describes the plant P in view of C_{i-1}. A new compensator C_i is constructed from $\hat{P}_i$ by the optimization of (2), which brings forth a small nominal performance norm $\|T(\hat{P}_i,C_i)\|_\infty$. The controller C_i is applied to the plant P, and new data is collected for the identification of the next nominal model.

In a straightforward application of the identification in (4) and the control design in (2) we encounter the following problem. As C_i is designed according to (2), this new compensator rests solely on the nominal model $\hat{P}_i$, and the new compensator C_i can be quite different from the old compensator C_{i-1}. In that case the performance degradation $\|T(P,C_i)-T(\hat{P}_i,C_i)\|_\infty$ is rather large, even though $T(\hat{P}_i,C_{i-1})$ is a good approximation of $T(P,C_{i-1})$, cf. (4). This implies that the resemblance between the feedback properties of $H(\hat{P}_i,C_i)$ and $H(P,C_i)$ is quite poor, despite the fact that C_i is optimally robust in view of the achieved nominal performance. In order to provide for a small performance degradation, we have to introduce weighting functions in the control design of (2).

In this note we simply use an adjustable scalar weight α_i. The controller C_i is designed according to

$$C_i = \arg\min_C \|T(\alpha_i\hat{P}_i, C/\alpha_i)\|_\infty. \quad (5)$$

This causes C_i to optimize robustness for a nominal performance level associated with α_i. The resulting designed feedback system has its bandwidth close to the cross-over frequency of $\alpha_i\hat{P}_i$ [11], and thus a large α_i corresponds to a high nominal performance. We intend to gradually increase the scalar design weight during the iteration in order to keep the performance degradation small at each iteration step. Eventually we reach a large design weight producing a high performance controller for the plant. The selection of appropriate design weights is discussed in Section 4.

The corresponding identification problem that has to be solved at each iteration step is

$$\hat{P}_i = \arg\min_{\bar{P}\in\mathcal{P}(\theta)} \|T(\alpha_{i-1}P, C_{i-1}/\alpha_{i-1}) - T(\alpha_{i-1}\bar{P}, C_{i-1}/\alpha_{i-1})\|_\infty. \quad (6)$$

As there exists no identification technique that can be used to solve (6), we replace the above H_∞ (or L_∞) approximation by an L_2 approximation. The rationale for this replacement is that the L_2 approximation yields a reasonably good nominal model in an L_∞ sense, provided that the error-term is sufficiently smooth. This observation is backed up by the result in [3] on the L_∞ consistency of L_2 estimators. The L_2-identification problem is discussed in the next section.

3 Control-relevant identification

We consider the feedback configuration of Fig. 2, in which the plant P is stabilized by the controller C_{i-1}. The feedback system is driven by the exogenous inputs r_1 and r_2 and the additive output noise v. The noise v is uncorrelated with r_1 and r_2 and it is modelled as $v = P_{yw}w$, where w is a white noise.

The problem of concern is to identify a nominal model $\hat{P}_i$ from measurements of u and y such that

$$\hat{P}_i = \arg\min_{\bar{P}\in\mathcal{P}(\theta)} \|T(P,C_{i-1}) - T(\bar{P},C_{i-1})\|_2. \quad (7)$$

This identification problem and the results below are readily extended to the case of $\alpha_i \neq 1$, cf. (6). We recall from the previous sections that we actually use system identification to find an approximate description of the feedback properties of $H(P,C_{i-1})$. Therefore we concentrate on the so-called "asymptotic bias distribution" due to undermodelling.

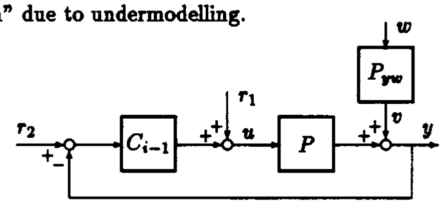

Fig. 2: Feedback configuration for identification

The set $\mathcal{P}(\theta)$ of candidate nominal models does not contain the plant, i.e. $P \notin \mathcal{P}(\theta)$. As a consequence the minimization of (7) from u and y combines all problems that are encountered in approximate identification and in closed-loop identification. The desired $\hat{P}_i$ cannot be derived by a direct application of some standard identification method to u and y (see [13] for a discussion). In order to obviate this problem we first represent the plant P by a right coprime factorization (definitions are provided in [17]).

The plant P is known to be stabilized by the latest controller C_{i-1}. As P belongs to the set of all systems that are stabilized by C_{i-1}, it can be represented by a coprime factorization that is dual to the (Youla-) parameterization of all stabilizing compensators [17]. We extend this dual parameterization to incorporate the "noise filter" P_{yw}. The proof is given elsewhere in these proceedings [16]. A similar parameterization has been used by Hansen [7] for closed-loop experiment design.

Theorem 3.1 ([16]) *Let an auxiliary model P_o be such that $H(P_o, C_{i-1})$ is stable. Further let P_o and C_{i-1} have right coprime factorizations (N_o, D_o) respectively (N_c, D_c). Then the feedback system of Fig. 2 is stable if and only if $[P_{yw} \;\; P]$ has a right coprime factorization*

$$\left[\, P_{yw} \;\; P \,\right] = \left[\, D_cS \;\; N_o + D_cR \,\right] \begin{bmatrix} I & 0 \\ N_cS & D_o - N_cR \end{bmatrix}^{-1} \tag{8}$$

where R and S are stable transfer functions.

This coprime factor representation of P and P_{yw} has been depicted in Fig. 3. The variable x appearing in this figure will be used for the identification of P. For notational convenience we define

$$N^a \doteq N_o + D_cR; \quad D^a \doteq D_o - N_cR, \tag{9}$$

so that $P = N^a(D^a)^{-1}$, which is the dual of the Youla parameterization.

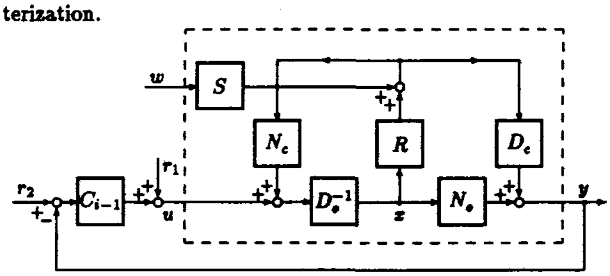

Fig. 3: Coprime factor representation of P and P_{yw}.

Lemma 3.2 *Let the assumptions of Theorem 3.1 hold. Then x of Fig. 3 can be reconstructed from u and y through*

$$x = (D_o + C_{i-1}N_o)^{-1}(u + C_{i-1}y). \tag{10}$$

The signal x is uncorrelated with w.

Proof: From Fig. 3 and with (9) we can write u and y as

$$u = D^a x - N_cSw; \quad y = N^a x + D_cSw. \tag{11}$$

Further, the equality

$$D_o + C_{i-1}N_o = D^a + C_{i-1}N^a \tag{12}$$

follows from replacing D^a and N^a with their definitions of (9). Substituting these expressions for u, y and $D_o + C_{i-1}N_o$ into the right hand side of (10) demonstrates the equivalence. Finally, from Fig. 3 we can express u as $u = r_1 + C_{i-1}r_2 - C_{i-1}y$ so that $u + C_{i-1}y$ equals $r_1 + C_{i-1}r_2$. Hence x is uncorrelated with w, because r_1 and r_2 are uncorrelated with w. □

Next we use x to estimate the frequency response of $T(P, C_{i-1})$.

Theorem 3.3 *Let the assumptions of Theorem 3.1 hold. Then an estimate of the frequency response of $T(P, C_{i-1})$ can be derived from u, y and C_{i-1}.*

Proof: By expressing P as $N^a(D^a)^{-1}$ and with (12) we can rewrite the transfer function $T(P, C_{i-1})$ to

$$T(P, C_{i-1}) = \begin{bmatrix} N^a \\ D^a \end{bmatrix} (D_o + C_{i-1}N_o)^{-1} \left[\, C_{i-1} \;\; I \,\right]. \tag{13}$$

As $(D_o + C_{i-1}N_o)$ and $[C_{i-1} \;\; I]$ are known, their frequency responses can be calculated. The frequency responses of N^a, D^a can be obtained directly from (11) with x as in Lemma 3.2. □

With Theorem 3.3 we have access to the frequency response of $T(P, C_{i-1})$ and thus $\hat{P}_i$ can be identified from (7). This minimization problem is all but trivial, because the nominal model $\hat{P}_i$ appears in $T(\hat{P}_i, C_{i-1})$ in a multiple and non-linear fashion. This optimization problem is attacked by the Newton-Raphson method in [14]. Due to its highly non-linear character the utility of this particular optimization hinges on a good initial estmate. In [14] such an estimate is obtained by identifying $\hat{P}_i$ in terms of its coprime factors.

4 Enhancement of the controller

The controller C_{i-1} has been obtained for some α_{i-1} by a minimization like in (5). Correspondingly, we use the method of the previous section to identify $\hat{P}_i$ as in (6) with $\|.\|_\infty$ replaced by $\|.\|_2$. Reverting to the discussion at the end of Section 2 we have to verify whether the minimal H_2-performance degradation actually involves a small H_∞-performance degradation. None of these norms can truly be determined, because we have only a finite number of frequency response samples over a restricted range of frequencies. Instead we evaluate both norms only for the available frequency response data. As the discussion below holds for both these (semi-)norms, we use the notation $\|.\|$ without a subscript.

In the light of (3) we require that the performance degradation $\|T(\alpha_{i-1}P, C_{i-1}/\alpha_{i-1}) - T(\alpha_{i-1}\hat{P}_i, C_{i-1}/\alpha_{i-1})\|$ is (much) smaller than the nominal performance norm $\|T(\alpha_{i-1}\hat{P}_i, C_{i-1}/\alpha_{i-1})\|$. (Otherwise a better $\hat{P}_i$ has to be determined, which might imply that a more complex modelset is needed.) Under this condition $H(\hat{P}_i, C_{i-1})$ provides a good description of $H(P, C_{i-1})$ in view of the weighted performance norms. We may expect that this holds also if C_{i-1} is slightly changed. Hence we design an improved controller C_i for $\hat{P}_i$ in such a way that C_i does not differ too much from the old controller C_{i-1}. Then C_i will achieve an improved performance for the plant P as well.

The change of the compensator will be moderate if the performance requirements are increased moderately. Hence we may choose α_i a bit larger than α_{i-1}. We outline how, in essence, this selection of α_i is guided by a frequency response estimate of P (details can be found in [14]). We build this estimate from the frequency response estimates of N^a and D^a used in the previous section. Then we evaluate the ratio of maximum singular values

$$\frac{\bar{\sigma}\{T(\alpha_i P, C_{i-1}/\alpha_i)(j\omega)\}}{\bar{\sigma}\{T(\alpha_i P, C_i/\alpha_i)(j\omega)\}} \Big/ \frac{\bar{\sigma}\{T(\alpha_i \hat{P}_i, C_{i-1}/\alpha_i)(j\omega)\}}{\bar{\sigma}\{T(\alpha_i \hat{P}_i, C_i/\alpha_i)(j\omega)\}}$$

and a similar ratio for the upper bound of (3). We choose α_i such that these ratios are just bounded by 0.7 and 1.3 for every frequency response sample of P. Thereby C_i changes $H(P, C_{i-1})$ similarly to $H(\hat{P}_i, C_{i-1})$.

As the choice of α_i is based on a "prediction" of the frequency response of $T(P, C_i)$, the feedback systems $H(\hat{P}_i, C_i)$ and $H(P, C_i)$ are expected to be similar in an L_∞-sense. We still have to ascertain the stability of the new control system $H(P, C_i)$ before the enhanced compensator C_i is actually applied to the plant P. This ascertainment of stability is necessary anyway, because the optimization of robustness against stable coprime factor perturbations is an unconstrained optimization[1], cf. (2) and (5). In ascertaining that $H(P, C_i)$ is stable, we use the frequency response estimates of N^a and D^a and a robustness margins that are conceived in terms of coprime factor perturbations. For details the interested reader is referred to [14, 16].

[1] With an unconstrained optimization there is no prior guarantee about the robustness that will be achieved.

5 Simulation Study

We apply the identification method of Section 3 and the controller enhancement of Section 4 to a simulation example. The data consist of 100 frequency response samples that are uniformly distributed over a logarithmic interval ranging from 0.1 to 100 rad/s. We use exact frequency response data in order that the effects due to undermodelling are not obscured by noise contributions. We merely list the results of this iterative high performance control design procedure, which is investigated in much more detail in [14].

The continuous-time plant P under investigation has a transfer function $n(s)/d(s)$ with

$$\begin{aligned} n(s) &= 30s^6+3020s^5+30538s^4+40373s^3+74041s^2+41972s+12467 \\ d(s) &= s^8+26.023s^7+321.70s^6+2635.9s^5+10412s^4 \\ &\quad +3091.4s^3+11032s^2+306.81s+986.86. \end{aligned}$$

In order to simulate a real application we pretend that the plant P is imprecisely known. Accordingly we do not use any knowledge of the plant's number of poles or (unstable) zeros; we just know that P is open-loop stable. Hence we cannot tell a priori how complex a compensator must be in order to obtain some performance. Conversely we do not know what performance is achievable with a compensator of constrained complexity.

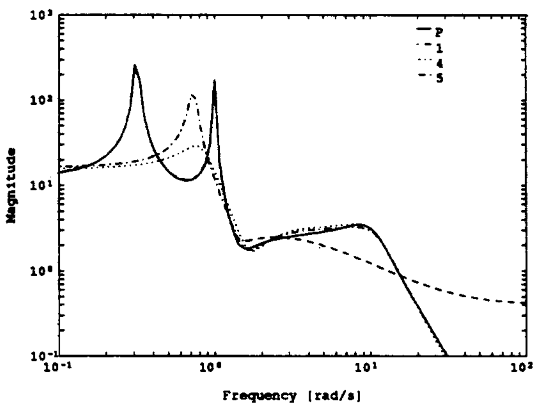

Fig. 4: Log-magnitudes of P (—), $\hat{P}_1$ (--), $\hat{P}_4$ (····) and $\hat{P}_5$ (-·-).

The iteration commences with an open-loop identification of $\hat{P}_1$. Fig. 4 shows the Bode log-magnitude plots of P (—) and $\hat{P}_1$ (--). The nominal model $\hat{P}_1$ provides an accurate description of the low frequency behavior of P. The mismatch at the higher frequencies hardly contributes to the identification criterion of (7) with $C_0=0$, because this criterion measures an additive error on a linear scale[2]. From $\hat{P}_1$ we design the compensator C_1 as in (5) with $\alpha_i=0.113$. We apply C_1 to P, we obtain new data, and we subsequently derive several nominal models and compensators. The iteration ends with the nominal model $\hat{P}_5(s)=\hat{n}(s)/\hat{d}(s)$, where

$$\begin{aligned} \hat{n}(s) &= 8.8\cdot 10^{-4}s^5-4.77\cdot 10^{-2}s^4+34.7s^3+2494s^2+1663s+6028 \\ \hat{d}(s) &= s^5+13.3s^4+156.3s^3+712.4s^2+131.3s+369.4, \end{aligned}$$

with the compensator $C_5(s)=n_c(s)/d_c(s)$, where

$$\begin{aligned} n_c(s) &= 71.407s^4+2182.1s^3+28718s^2+23854s+68457 \\ d_c(s) &= s^4+129.16s^3+4829.0s^2+3344.1s+11571, \end{aligned}$$

and with $\alpha_5=20$. The evolution of the nominal models and of the controllers are illustrated respectively in Fig. 4 and Fig. 5.

[2]We want a small "additive" performance degradation measured on a linear scale, cf. (3). Hence a minimization of some relative error is inappropriate.

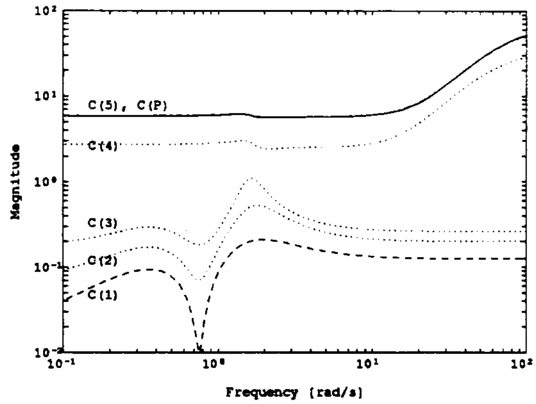

Fig. 5: Log-magnitudes of the designed controllers.

The latter figure displays the gradual increase of control action. The former figure reveals that during the iteration the accuracy of the nominal model is improved in the high frequency range at the expense of a large mismatch for the lower frequencies. Despite the large open-loop mismatch between P and $\hat{P}_5$ (see again Fig. 4), the nominal model $\hat{P}_5$ is suited for high performance control design. This is illustrated in Fig. 6, which shows the

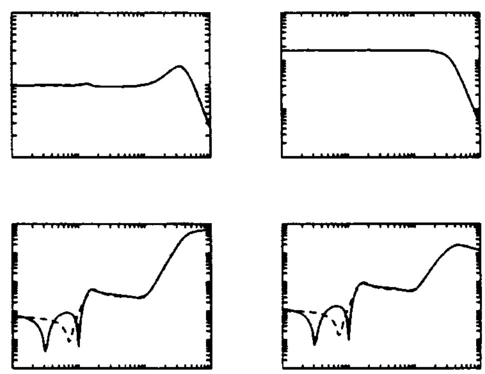

Fig. 6: Log-magnitudes of $T(P,C_5)$ (—) and $T(\hat{P}_5,C_5)$ (--).

log-magnitudes of $T(P,C_5)$ and of $T(\hat{P}_5,C_5)$. Considering the logarithmic scale we may conclude that P and $\hat{P}_5$ have very similar high performances under feedback by C_5. Hence the couple $\hat{P}_5,C_5$ is a solution to the joint problem of approximate identification and model-based control design.

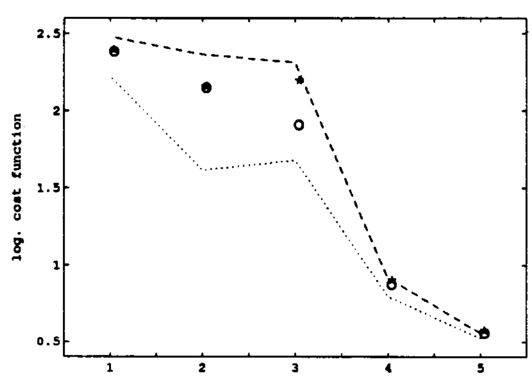

Fig. 7: Logarithm of the performance norms.

We evaluate the performance norms for all pairs of nomin models and compensators in regard of α_5 as explained in Section 4. That is, we determine for instance $\|T(\alpha_5 P, C_i/\alpha_5)\|$ as the maximum singular value over all frequency reponse samples. These performance norms have been plotted in Fig. 7. The indices at the horizontal axis indicate the iteration step. The performance norms corresponding to $T(\hat{P}_i, C_i)$ and $T(P, C_i)$ are marked respectively by 'o' and '$\star$'. The upper bound of (3), indicated by (--), and the analogous lower bound ($\cdots$) disclose that the approximation of $T(P, C_i)$ by $T(P_i, C_i)$ is relatively accurate. This is a direct consequence of the frequency reponse based controller enhancement of Section 4. The figure also displays that the "worst-case" performance (--) is improved in each step of the iteration. Finally Fig. 8 shows the evolution of the sensitivity that is achieved for the plant P.

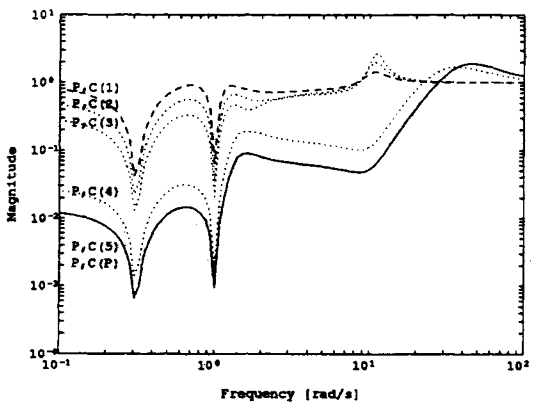

Fig. 8: Sensitivities achieved for the plant P.

We complete the evaluation by using the method of (5) and α_5 to design also the compensator C_P of order 4 directly from the plant P. In regard of α_5 this C_P is the optimal compensator of order 4 that can be designed for and from the plant P. In Fig. 5 we see that the frequency responses of C_P and C_5 are indiscernible, which produces indiscernible sensitivities for P (see Fig. 8). Thus the iteratively designed high performance compensator C_5 is almost identical with the optimal plant-based compensator C_P, even though no exact knowledge of P nor any information of C_P has been used to achieve C_5.

Lastly we elucidate the need of an iteration to solve the joint problem of approximate identification and model-based control design. The left upper term of $T(P, C_5) - T(\hat{P}_5, C_5)$, which equals $PC_5(I + PC_5)^{-1} - \hat{P}_5 C_5(I + \hat{P}_5 C_5)^{-1}$, can be rewritten to $(I + PC_5)^{-1}(P - \hat{P}_5)C_5(I + \hat{P}_5 C_5)^{-1}$. Similar expressions can be derived for the other elements of $T(P, C_5)$ and $T(\hat{P}_5, C_5)$. Hence $\hat{P}_5, C_5$ make a *couple* that produces a small mismatch

$$W_L(P, C_5)\ (P - \hat{P}_5)\ W_R(\hat{P}_5, C_5),$$

where W_L and W_R are weighting functions depending on P, $\hat{P}_5$ and C_5. It is tempting to suggest that $\hat{P}_5$ could have been obtained directly from a weighted open-loop identification. However, $W_L(P, C_5)$ and $W_R(\hat{P}_5, C_5)$ depend on the outcome of the iteration, and thus the required weighting functions are not available at the outset.

6 Concluding remarks

We addressed the problem of designing a high performance compensator for an imprecisely known plant. We tackled this problem by an iterative scheme of repeated identification and control design. At each stage of the iteration data is obtained from the plant while it is controlled by the latest compensator. The task of the identification is to model the current feedback properties of the controlled plant. The resulting nominal model is used to slightly improve the nominal performance, so that the imprecisely known plant's performance is improved as well. As the iterative design procedure evolves, it learns about the control-relevant dynamics of the plant in question. The resulting nominal model is accurate near the cross-over frequency and, at least as important, the large mismatch at other frequencies does not impair the control design. In addition the iteration reveals the performance that is attainable for the imprecisely known plant.

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