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Identification of normalized coprime plant factors for iterative model and controller enhancement

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Abstract

Recently introduced methods of iterative identification and control design are directed towards the design of high performing and robust control systems. These methods show the necessity of identifying approximate models from closed loop plant experiments. In this paper a method is proposed to approximately identify normalized coprime plant factors from closed loop data. The fact that *normalized* plant factors are estimated gives specific advantages both from an identification and from a robust control design point of view. It will be shown that the proposed method leads to identified models that are specifically accurate around the bandwidth of the closed loop system. The identification procedure fits very naturally into the iterative identification/control design scheme as presented in [15].

1 Introduction

Recently it has been motivated that the problem of designing a high performance control system for a plant with unknown dynamics through separate stages of (approximate) identification and model based control design requires iterative schemes to solve the problem [10, 15, 17, 24]. In these iterative schemes each identification is based on new data collected while the plant is controlled by the latest compensator. Each new nominal model is used to design an improved compensator, which replaces the old compensator, in order to improve the performance of the controlled plant.

A few iterative schemes proposed in literature have been based on the prediction error identification method, together with LQG control design [24, 7]. In [15, 16, 10] iterative schemes have been worked out, employing a Youla parametrization of the plant, and thus dealing with coprime factorizations in both identification and control design stage; as control design methods a robustness optimization procedure of [12, 3] is applied in [15, 16], while in [10] the IMC-design method is employed. For a general background and a more extensive overview and comparison of different iterative schemes the reader is referred to [6, 1].

One of the central aspects in almost all iterative schemes is the fact that the identification of a control-relevant plant model has to be performed under closed loop experimental conditions. Standard identification methods have not been able to provide satisfactory models for plants operating in closed loop, except for the case that input/output dynamics and noise characterictics can be modelled exactly.

Recently introduced approaches to the closed loop identification problem [8, 14, 10, 15, 20] show the possibility of also identifying approximate models, where the approximation criterion (if the number of data tends to infinity) becomes explicit, i.e. it becomes independent of the - unknown - noise disturbance on the data. This has opened the possibility to identify approximate models from closed loop data, where the approximation criterion explicitly can be "controlled" by the user, despite a lack of knowledge about the noise characteristics. In the corresponding iterative schemes of identification and control design this approximation criterion then is tuned to generate a control-relevant plant model. The identification methods considered in the iterative procedures presented in [15, 16, 10] employ a plant representation in terms of a coprime factorization $P = ND^{-1}$, while in [15, 16] the two plant factors N, D are separately identified from closed-loop data.

Coprime factor plant descriptions play an important role in control theory. The parametrization of the set of all controllers that stabilize a given plant greatly facilitates the design of controllers [22]. The special class of *normalized* coprime factorizations has its applications in design methods [12, 3] and robustness margins [21, 5, 13]. If we have only plant input-output data at our disposal, then a relevant question becomes how to model the normalized coprime plant factors as good as possible. In this paper we will focus on the problem of identifying normalized coprime plant factors on the basis of closed loop experimental data.

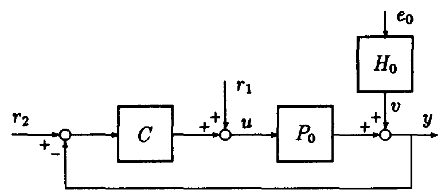

Fig. 1: Feedback configuration

As an experimental situation we will consider the feedback configuration as depicted in Fig. 1, where P_0 is an LTI-(linear time-invariant), possibly unstable plant, H_0 a stable LTI disturbance filter, e_0 a sequence of identically distributed independent random variables and C an LTI-(possibly unstable) controller. The external signals r_1,r_2 can either be considered as external reference (setpoint) signals, or as (unmeasurable) disturbances. In general we will assume to have available only measurements of the input and output signals u and y, and knowledge of the controller C that has been implemented. We will also regularly refer to the artificial signal $r(t) := r_1(t) + Cr_2(t)$.

First we will discuss some preliminaries about normalized coprime factorizations and their relevance in control design. In section 3 a generalized framework is presented for closed loop identification of coprime factorizations. Next we present a multi-step algorithm for identification of normalized factors. In section 5 we briefly show the experimental results that were obtained when applying the algorithm to the radial servo-mechanism in a Compact Disc player.

$\mathcal{RH}_\infty$ will denote the set of real rational transfer functions in $\mathcal{H}_\infty$, analytic on and outside the unit circle; $\mathbb{R}[z^{-1}]$ is the ring of (finite degree) polynomials in the indeterminate z^{-1} and q is the forward shift operator: $qu(t) = u(t+1)$.

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2 Normalized coprime factorizations

Consider a LTI system P, then P has a *right coprime factorization* (r.c.f.) (N, D) over $\mathcal{RH}_\infty$ if there exist $U, V, N, D \in \mathcal{RH}_\infty$ such that

$$P(z) = N(z)D^{-1}(z); \quad UN + VD = I. \tag{1}$$

In addition a right coprime factorization (N_n, D_n) of P is called *normalized* if it satisfies

$$N_n^T(z^{-1})N_n(z) + D_n^T(z^{-1})D_n(z) = I. \tag{2}$$

Dual definitions exist for left coprime factorizations (l.c.f.).
One of the properties of normalized coprime factors is that they form a decomposition of the system P in minimal order (stable) factors. In other words, if the plant has McMillan degree n_p, then normalized coprime factors of P will also have McMillan degree n_p[1]. Additionally there will always exist polynomials $a, b, f \in \mathbb{R}[z^{-1}]$ of degree n_p such that $N_n = f(z^{-1})^{-1}b(z^{-1})$ and $D_n = f(z^{-1})^{-1}a(z^{-1})$.
In robust stability analysis normalized coprime factors play an important role, reflected in the following robustness result [12, 3]. Let $\hat{P}$ be a plant model that is stabilized by the controller C. Moreover let (N_n, D_n) be a normalized r.f.c. of $\hat{P}$, and let the real plant P_0 be such that there exist stable perturbations Δ_N, Δ_D such that $P_0 = (N_n + \Delta_N)(D_n + \Delta_D)^{-1}$.
Then C stabilizes the plant P_0 for all Δ_N, $\Delta_D \in \mathcal{RH}_\infty$ satisfying $\left\| \begin{matrix} \Delta_N \\ \Delta_D \end{matrix} \right\|_\infty < \gamma$ if and only if $\gamma \leq \|T(\hat{P}, C_{\hat{P}})\|_\infty^{-1}$, with $T(\hat{P}, C) := \begin{bmatrix} \hat{P} \\ I \end{bmatrix} [I + C\hat{P}]^{-1} \begin{bmatrix} C & I \end{bmatrix}$.
This result shows that when we would have access to the normalized coprime factors of the plant, together with an error bound on these (estimated) factors (in the form of error bounds on the mismatches Δ_N and Δ_D), then immediate results follow for the robust stability of the plant.
This result may not seem to be too striking, since a similar situation can be reached by any hard-bounded uncertainty on the system's transfer function, and application of the small gain theorem. However uncertainty decriptions in normalized coprime factor form have been shown to have some specific advantages, as the ability to deal with unstable plants and their close connection with uncertainty descriptions in the gap-metric [5].
The control design method of [3, 12] is directed towards optimizing this same robustness margin as discussed above. This control design method is employed in the iterative identification/control design scheme of [15, 16].

3 Closed loop identification of coprime factorizations

3.1 Closed loop identification

The closed loop identification problem is not straighforwardly solvable in the case that one is not sure that exact models of the plant and its disturbances can be obtained in the form of a consistent estimate of P_0 and H_0. What we would like to find - based on signal measurements - is a model $\hat{P}$ of the plant P_0 such that there exists an explicit approximation criterion $J(P_0, \hat{P})$ indicating the way in which P_0 has been approximated (at least asymptotically in the number of data), while $J(P_0, \hat{P})$ is independent of the unknown noise disturbance on the data.
Additionally one would like to be able to tune this approximation criterion to get an approximation of P_0 that is desirable in view of the control design to be performed. This explicit tuning of the approximation criterion is possible within the classical framework only when open-loop experiments can be performed.
Let's consider a few alternatives to deal with this closed-loop approximate identification problem, assuming the signal r is available from measurements[2]:

- If we know the controller C, we could do the following: Consider a parametrized model $P(\theta)$, $\theta \in \Theta$, and identify θ through:

$$y(t) = \frac{P(\theta)}{1 + P(\theta)C} r(t) + \varepsilon(t) \tag{3}$$

  by least squares minimization of the prediction error $\varepsilon(t)$.

  This first alternative leads to a complicatedly parametrized model set, and as a result it is not attractive, although it provides us with a consistent estimate of P irrespective of the noise modelling, and with an explicit approximation criterion.

- Identify transfer functions

$$H_{yr} = \frac{P}{1 + PC} \quad \text{and} \quad H_{ur} = \frac{1}{1 + PC}$$

  as black box transfer functions $\hat{H}_{yr}, \hat{H}_{ur}$, then an estimate of P can be obtained as $\hat{P} = \hat{H}_{yr}\hat{H}_{ur}^{-1}$.

  This method shows a decomposition of the problem in two parts, actually decomposing the system into two separate (high) order factors, sensitivity function and plant-times-sensitivity function. In this setting it will be hard to "control" the order of the model to be identified, as the quotient of the two estimated transfer functions $\hat{H}_{yr}$, $\hat{H}_{ur}$ will generally not cancel the common dynamics that are present in both functions. As a result the model order will become unnecessarily high.

- As a third alternative we can first identify H_{ur} as a black box transfer function $\hat{H}_{ur}$, and consecutively identify P from:

$$y(t) = P(\theta)\hat{u}_r(t) + \varepsilon(t) \quad \text{with } \hat{u}_r(t) := \hat{H}_{ur} r(t).$$

  This method is presented in [20]. It also uses a decomposition of the plant P in two factors as in the previous method, now requiring a very accurate estimate of H_{ur} in the first step. An explicit approximation criterion can be formulated.

If, as in the last two methods, the plant is represented as a quotient of two factors of which estimates can be obtained from data, it is advantageous to let these factors have the minimal order, thus avoiding the problem that the resulting plant model has an excessive order, caused by non-cancelling terms.

3.2 A generalized framework

We will now present a generalized framework for identification of coprime plant factors from closed loop data. It will be shown to have close connections to the Youla-parametrization, as employed in the identification schemes as proposed in [8, 14, 15, 10].
Let us consider the notation[3]

$$S_0(z) = (I + C(z)P_0(z))^{-1} \quad \text{and} \tag{4}$$
$$W_0(z) = (I + P_0(z)C(z))^{-1}. \tag{5}$$

[1] In the exceptional case that P contains all-pass factors, (one of) the normalized coprime factors will have McMillan degree $< n_p$, see [19].

[2] Similar results follow if either r_1 or r_2 are available from measurements.

[3] The main part of the paper is directed towards multivariable systems, and so we distinguish between output and input sensitivity.

Then we can write the system's equations as[4]

$$y(t) = P_0(q)S_0(q)r(t) + W_0(q)H_0(q)e_0(t) \quad (6)$$
$$u(t) = S_0(q)r(t) - C(q)W_0(q)H_0(q)e_0(t). \quad (7)$$

Note also that

$$r(t) = r_1(t) + C(q)r_2(t) = u(t) + C(q)y(t). \quad (8)$$

Using knowledge of $C(q)$, together with measurements of u and y, we can simply "reconstruct" the reference signal r in (8) So in stead of a measurable signal r, we can equally well deal with the situation that y, u are measurable and C is known.

It can easily be verified from (6),(7) that the signal $\{u(t) + C(q)y(t)\}$ is uncorrelated with $\{e_0(t)\}$ provided that r is uncorrelated with e_0. This shows with equations (6),(7) that the identification problem of identifying the transfer function from signal r to $(y,u)^T$ is an "open loop"-type of identification problem, since r is uncorrelated with the noise terms dependent on e_0. The corresponding factorization of P_0 that can be estimated in this way is the factorization (P_0S_0, S_0), i.e. $P_0 = (P_0S_0) \cdot S_0^{-1}$, as also employed in e.g. [25].

However this is only one of the many factorizations that can be identified from closed loop data in this way. By introducing an auxiliary signal

$$x(t) := F(q)r(t) = F(q)(u(t) + C(q)y(t)) \quad (9)$$

with $F(z)$ a fixed stable rational transfer function, we can rewrite the system's relations as

$$y(t) = P_0(q)S_0(q)F(q)^{-1}x(t) + W_0(q)H_0(q)e_0(t) \quad (10)$$
$$u(t) = S_0(q)F(q)^{-1}x(t) - C(q)W_0(q)H_0(q)e_0(t), \quad (11)$$

and thus we have obtained another factorization of P_0 in terms of the factors $(P_0S_0F^{-1}, S_0F^{-1})$. Since we can reconstruct the signal x from measurement data, these factors can also be identified from data, as in the situation considered above, provided of course that the factors themselves are stable. We will now characterize the freedom that is present in choosing this filter F.

Proposition 3.1 *Consider a data generating system according to* (6),(7)*, such that C stabilizes P_0, and let $F(z)$ be a rational transfer function defining*

$$x(t) = F(q)(u(t) + C(q)y(t)). \quad (12)$$

Let the controller C have a left coprime factorization $(\tilde{D}_c, \tilde{N}_c)$. Then the following two expressions are equivalent

a. the mappings $col(r_2, r_1) \rightarrow x$ and $x \rightarrow col(y,u)$ are stable;

b. $F(z) = W\tilde{D}_c$ with W any stable and stably invertible rational transfer function. □

The proof of this Proposition is added in the appendix.

Note that stability of the mappings mentioned under (a) is required in order to guarantee that we obtain a bounded signal x as an input in our identification procedure, and that the factors to be estimated are stable, so we are able to apply the standard (open-loop) prediction error methods and analysis thereof.

Note that all factorizations of P_0 that are induced by these different choices of F reflect factorizations of which the stable factors can be identified from input/output data, cf. equations (10),(11).

The construction of the signal x is schematically depicted in Figure 2. Note that we have employed (8) which clearly shows that x is uncorrelated with e_0 provided the external signals are also uncorrelated with e_0.

For any choice of F satisfying the conditions of Proposition 3.1 the induced factorization of P_0 is right coprime, as shown next.

[4]Note that we have employed the relations $W_0P_0 = P_0S_0$ and $S_0C = CW_0$.

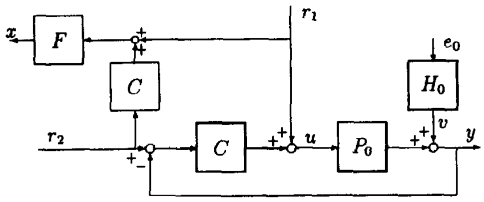

Fig. 2: Construction of auxiliary signal x from closed loop data.

Proposition 3.2 *Consider the situation of Proposition 3.1. For any choice of $F = W\tilde{D}_c$ with W stable and stably invertible, the induced factorization of P_0, given by $(P_0S_0F^{-1}, S_0F^{-1})$ is right coprime.* □

Proof: Let (X,Y) be right Bezout factors of (N,D), and denote $[X_1 \; Y_1] = W(\tilde{D}_cD + \tilde{N}_cN)[X \; Y]$. Then by employing (A.1) it can simply be verified that X_1, Y_1 are stable and are right Bezout factors of $(P_0S_0F^{-1}, S_0F^{-1})$. □

We will employ the freedom in the filter F, in order to tune the specific coprime factors that can be estimated from closed loop data. Similar to the Youla parametrization, we will use an auxiliary model P_x that is required to be stabilized by C.

Proposition 3.3 *Consider the situation of Propositions 3.1,3.2. Let P_x be an auxiliary model with r.c.f. (N_x, D_x) that is stabilized by C, which has l.c.f. $(\tilde{D}_c, \tilde{N}_c)$. Then a valid choice of W (satisfying (b) in Proposition 3.1) is given by $(\tilde{D}_cD_x + \tilde{N}_cN_x)^{-1}$, and the induced right coprime factorization of P_0 is given by*

$$N_0 = P_0(I + CP_0)^{-1}(I + CP_x)D_x \quad (13)$$
$$D_0 = (I + CP_0)^{-1}(I + CP_x)D_x. \quad (14)$$

Proof: With Lemma A.1 it follows that $\tilde{D}_cD_x + \tilde{N}_cN_x$ is stable and stably invertible, and thus it is an appropriate choice for W^{-1}. The resulting N_0 and D_0 follow by simple substitution of $F = W\tilde{D}_c = (\tilde{D}_cD_x + \tilde{N}_cN_x)^{-1}\tilde{D}_c = (D_x + CN_x)^{-1}$. □

Note that for any given controller C, and any stable and stably invertible W, there always exists an auxiliary model P_x that satisfies $(\tilde{D}_cD_x + \tilde{N}_cN_x)^{-1} = W$. This implies that the freedom that is present in W, as shown in Proposition 3.1, is not restricted by the specific choice of W in Proposition 3.3.

The representation of P_0 in terms of the coprime factorization above, shows great resemblance with the dual Youla-parametrization, i.e. the parametrization of all plants that are stabilized by a given controller. This connection is shown next.

Proposition 3.4 *Schrama (1992). Let C be a controller with r.c.f. (N_c, D_c), and let P_x with r.c.f. (N_x, D_x) be any system that is stabilized by C. Then*

(a) A system P_0 is stabilized by C if and only if there exists a stable R satisfying

$$N_x + D_cR = P_0(I + CP_0)^{-1}(I + CP_x)D_x \quad (15)$$
$$D_x - N_cR = (I + CP_0)^{-1}(I + CP_x)D_x. \quad (16)$$

(b) The stable matrix R in (a) is uniquely determined by

$$R = D_c^{-1}(I + P_0C)^{-1}(P_0 - P_x)D_x. \quad (17)$$

The proposition shows that the dual Youla-parametrization induces a set of coprime factorizations (15),(16) that have exactly the same structure as the coprime factorizations that can be identified from closed loop data, with an appropriate choice of the data filter F.
In the next section we will show how we can exploit the freedom in choosing F, N_x and D_x in order to arrive at an estimate of *normalized* coprime factors of the plant.

4 An algorithm for identification of normalized coprime factors

The idea of arriving at normalized coprime factorizations of P_0 is based on the following observation. Consider the coprime factors (13),(14) that are accessible from closed loop data as discussed before. Suppose we can find an auxiliary model P_x that is an accurate (possibly high order) approximation of the plant P_0, and we construct a normalized r.c.f. (N_n, D_n) of P_x. Using these normalized r.c.f.'s as N_x and D_x in (13),(14), it follows with (15),(16) that $N_0 = N_n + D_cR$ and $D_0 = D_n - N_cR$. Employing $P_x \approx P_0$ which leads to $R \approx 0$ then shows that (N_0, D_0) (approximately) equals a normalized r.c.f. of P_0. This line of thought is formalized in the following algorithm

1. Let there be available a nominal model P_{nom} of the plant P_0, such that P_{nom} is stabilized by C. Set $P_x = P_{nom}$ and construct a r.c.f. $P_x = N_x D_x^{-1}$. Construct the data filter F according to Proposition 3.3:

$$F = D_x^{-1}(I + CP_x)^{-1} \tag{18}$$

   and use this data filter to construct an auxiliary signal $x = F(u + Cy)$. The corresponding closed loop system equations become

$$\begin{aligned} y(t) &= N_0 x(t) + W_0 H_0 e_o(t) && (19)\\ u(t) &= D_0 x(t) - CW_0 H_0 e_0(t) && (20) \end{aligned}$$

   with N_0, D_0 given by (13),(14).

2. Use the signals (y, u, x) in a (least squares) identification algorithm with a output error model structure ([11]):

$$\varepsilon(t,\theta) = \begin{pmatrix} y(t) \\ u(t) \end{pmatrix} - \begin{bmatrix} N(\theta) \\ D(\theta) \end{bmatrix} x(t) \tag{21}$$

   considering (y, u) as output signals and x as input signal.

   Use this parametrization to identify the coprime factors N_0, D_0 as accurately as possible through high-order modelling, e.g. by employing orthogonal basis functions in a linear regression scheme. In this respect the new method of constructing orthogonal basis functions that contain system dynamics shows very promissing results, see [9], as also applied for identification purposes in [4].

   This step is comparable to the first step in the so-called two-stage identifcation procedure in [20]. The identified coprime factors are denoted as $\hat{N}$, $\hat{D}$.

3. Denote $P_n := \hat{N}\hat{D}^{-1}$ and construct a normalized right coprime factorization (N_n, D_n) such that $P_n = N_n D_n^{-1}$. A procedure for constructing this normalized r.c.f. can be found in [23, 2]. Set $P_x = P_n$, $D_x = D_n$, $N_x = N_n$.

4. Construct a new data filter F according to (18) and generate a new auxiliary signal $x = F(u + Cy)$. The corresponding system's equations are again given by (19),(20).

   Employing the results of Proposition 3.4 it follows that

$$\begin{aligned} N_0 &= N_n + D_cR && (22)\\ D_0 &= D_n - N_cR, && (23) \end{aligned}$$

   while (17) shows that when P_n approaches P_0, then R will approach 0 and the above equations show that the coprime factors N_0, D_0 that can be estimated from closed loop data are "almost normalized".

5. Now again identify coprime plant factors as in Step 2, using measurement signals (y, u, x) and an output error model structure (21) where $N(\theta)$ and $D(\theta)$ are parametrized as

$$\begin{aligned} N(\theta) &= f(q^{-1},\theta)^{-1} b(q^{-1},\theta) && (24)\\ D(\theta) &= f(q^{-1},\theta)^{-1} a(q^{-1},\theta) && (25) \end{aligned}$$

   with a, b and f (matrix) polynomials of specified degree, having coefficients that are collected in the parameter vector θ. This parametrization, where N and D have a common denominator, guarantees that the McMillan degree of the ultimately identified model is equal to the McMillan degree of the estimated coprime factors.

   The parameter estimate is obtained by

$$\hat{\theta}_N = \arg\min_\theta \frac{1}{N}\sum_{t=1}^{N} \varepsilon_f^T(t,\theta)\varepsilon_f(t,\theta), \tag{26}$$

   with $\varepsilon_f(t,\theta) = L\varepsilon(t,\theta)$, and $L \in \mathcal{RH}_\infty^{(n_y+n_u)\times(n_y+n_u)}$, decomposed as $L = diag(L_y, L_u)$.

6. The result of the algorithm is composed of estimated (almost normalized) right coprime plant factors $(N(\hat{\theta}_N), D(\hat{\theta}_N))$ and a resulting plant model $P(\hat{\theta}_N) = N(\hat{\theta}_N)D(\hat{\theta}_N)^{-1}$.

As shown in the previous section, the plant coprime factorizations that are accessible from closed experimental data are determined by the specific choice of filter F and signal x that are chosen. The coprime factorizations that can be obtained in this way can be made exactly normalized only in the situation that we have exact knowledge of the plant P_0. In the algorithm presented above, we have replaced this exact knowledge of P_0 by a (very) high order accurate estimate of P_0. This knowledge is used to shape the specific set of coprime factors that is accessible from data.
The nominal model P_{nom} that the algorithm is started by, can be obtained from previous experiments on the plant, or from the previous iteration step in an iterative identification/control design procedure. Note that the order of the "high order" estimate of P_0 in step 2 may be strongly dependent on the nominal model P_{nom} that is used as an auxiliary model in the first step. The more accurate this auxiliary model, the more common dynamics is cancelled in the coprime factors (13),(14), and consequently the easier N_0, D_0 can be accurately described by a model of limited order. This motivates an iterative repetition of steps 1–3 in the algorithm presented above, in which the high order normalized r.c.f.'s in step 3 are used as auxiliary factors again in step 1 of the procedure, thus generating a new signal x to be used again for identification. Such an iterative procedure has also been applied in the application example discussed in the next section.
In order to explicitly write down the asymptotic identification criterion that has been minimized in the last step, note that we can write

$$\varepsilon(t,\theta) = \begin{bmatrix} L_y(N_0 - N(\theta)) \\ L_u(D_0 - D(\theta)) \end{bmatrix} x(t) + \begin{bmatrix} W_0 H_0 \\ -CW_0 H_0 \end{bmatrix} e_0(t) \tag{27}$$

with N_0, D_0 given by (22),(23). As a result the asymptotic parameter estimate $\theta^* = \text{plim}_{N\to\infty} \hat{\theta}_N$ is characterized by

$$\theta^* = \arg\min_{\theta} \int_{-\pi}^{\pi} \left[|N_0(e^{i\omega}) - N(e^{i\omega},\theta)|^2 |L_y(e^{i\omega})|^2 + \right.$$
$$\left. + |D_0(e^{i\omega}) - D(e^{i\omega},\theta)|^2 |L_u(e^{i\omega})|^2 \right] \cdot \Phi_x(\omega) d\omega \quad (28)$$

with $x(t) = D_n^{-1}(I + CP_n)^{-1}[u(t) + C(q)y(t)]$ and N_0, D_0 given by (22),(23).
If the first identification step (Step 1) of identifying (N, D) is accurately enough ($P_n \to P_0$), then N_n, D_n tend to be normalized right coprime factorizations of the plant. Since $P_x = P_n$, applying (17) shows that $R \to 0$, and the R-dependent terms in (22),(23) will vanish. The resulting frequency-domain expression shows that we obtain a (frequency-weighted) LS-approximation of normalized rcf's of the plant. The type of frequency-weighting can be influenced by designing the spectrum of the reference signal r and by appropriate prefilter L.
Note that in this identification method there is no additional problem if the plant and/or controller are unstable.

5 Application to a mechanical servo system

We will illustrate the proposed identification algorithm by applying it to data obtained from experiments on the radial servo mechanism in a CD (compact disc) player. For a more extensive description of this servo mechanism we refer to [18, 4]. The radial servo mechanism concerns an unstable system, due to a double integrator. In the present configuration the radial control loop has been realized by a controller which consists of a lead–lag element and proportional and integrating action.
This experimental set up is used to gather time sequences of $u(t)$ and $y(t)$ in the radial control loop, exciting the signal $r_1(t)$. The signals were sampled at 25kHz and the reference signal $r_1(t)$ was chosen to be a bandlimited white noise signal in the frequency domain of interest (100Hz–10kHz).
Results of applying the algorithm presented in section 4 are shown in a couple of figures. Figure 3 shows the result of the estimated coprime factors $\hat{N}$, $\hat{D}$ at step 2 of the algorithm. This is the high-order estimate, being the result of a number of iterations over steps 1–3 as mentioned before. Order of the models is 24. The results are compared with non-parametric spectral estimates of the corresponding plant factors.
Figure 4 shows the final result, estimated low-order coprime factors, with model order 10. In Figure 5 it is checked whether the finally obtained estimates $N(\hat{\theta}_N), D(\hat{\theta}_N)$ indeed are normalized. To this end we have plotted the frequency response of $N^T(z^{-1},\hat{\theta}_N)N(z,\hat{\theta}_N) + D^T(z^{-1},\hat{\theta}_N)D(z,\hat{\theta}_N)$ and the same response of the high order (unnormalized) estimates.
Note that the control-relevant frequency region, i.e. the area of the cross-over frequency, is very well represented in both normalized coprime factors. I.e. the dynamics that is related to this frequency region is relatively easy to be identified from these factors.

Conclusions

In this paper it is shown that it is possible to identify (almost) normalized coprime plant factors based on closed loop experiments. A general framework is given for closed loop identification of coprime factorizations, and it is shown that the freedom that is present in generating appropriate signals for identification can be exploited to obtain (almost) normalized coprime plant factors from closed loop data. The resulting multi-step algorithm is illustrated with results that are obtained from closed loop experiments on an open loop unstable mechanical servo system.

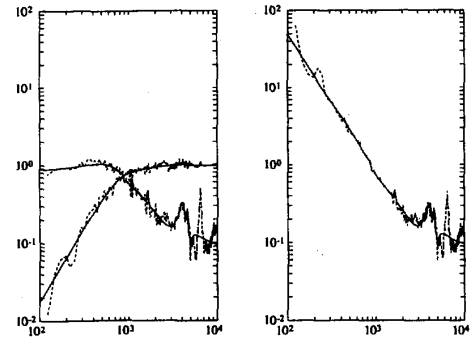

Fig. 3: Bode magnitude plots of high order model (step 2).
a: Identified coprime plant factors $\hat{N}$, $\hat{D}$ of 24th order model (solid line), and spectral estimates of the same factors (dashed line).
b: Estimated plant model $\hat{N}\hat{D}^{-1}$ (solid line) and spectral estimate (dashed line).

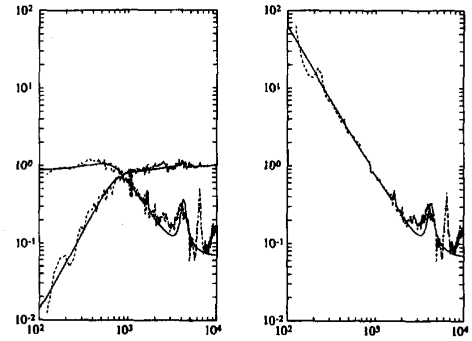

Fig. 4: Bode magnitude plots of final 10-th order model.
a: Identified coprime plant factors $N(\hat{\theta}_N)$, $D(\hat{\theta}_N)$ (solid line), and spectral estimates of the same factors (dashed line).
b: Estimated plant model $N(\hat{\theta}_N)D(\hat{\theta}_N)^{-1}$ (solid line) and spectral estimate (dashed line).

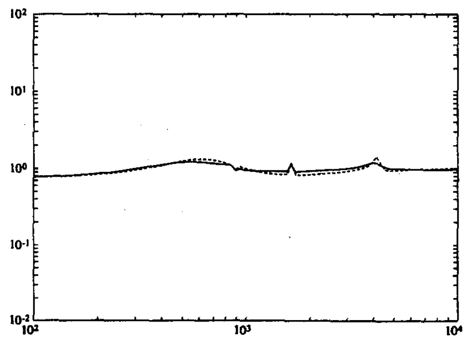

Fig. 5: Bode magnitude plot of $\hat{N}^T(z^{-1})\hat{N}(z) + \hat{D}^T(z^{-1})\hat{D}(z)$ (solid line) and $N^T(z^{-1},\hat{\theta}_N)N(z,\hat{\theta}_N)+$ $+D^T(z^{-1},\hat{\theta}_N)D(z,\hat{\theta}_N)$ (dashed line).

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Appendix

Lemma A.1 [22]. *Consider rational transfer functions $G_0(z)$ with right coprime factorization (N, D) and $C(z)$ with left coprime factorization $(\tilde{D}_c, \tilde{N}_c)$. Then $T(G_0, C) = \begin{bmatrix} G_0 \\ I \end{bmatrix} (I + CG_0)^{-1} \begin{bmatrix} C & I \end{bmatrix}$ is stable if and only if $\tilde{D}_c D + \tilde{N}_c N$ is stable and stably invertible.* □

Proof of Proposition 3.1.

$(a) \Rightarrow (b)$. By writing $\begin{bmatrix} G_0 S_0 \\ S_0 \end{bmatrix} = \begin{bmatrix} G_0 \\ I \end{bmatrix} (I + CG_0)^{-1}$ and substituting a right coprime factorization (N, D) for G_0, and a left coprime factorization $(\tilde{D}_c, \tilde{N}_c)$ for C we get, after some manipulation, that

$$\begin{bmatrix} G_0 S_0 \\ S_0 \end{bmatrix} = \begin{bmatrix} N \\ D \end{bmatrix} (\tilde{D}_c D + \tilde{N}_c N)^{-1} \tilde{D}_c \qquad \text{(A.1)}$$

and stability of $\begin{bmatrix} G_0 S_0 F^{-1} \\ S_0 F^{-1} \end{bmatrix}$ is equivalent with stability of $\begin{bmatrix} N \\ D \end{bmatrix} (\tilde{D}_c D + \tilde{N}_c N)^{-1} \tilde{D}_c F^{-1}$. Premultiplication of the latter expression with the stable transfer function $(\tilde{D}_c D + \tilde{N}_c N) \begin{bmatrix} X & Y \end{bmatrix}$ with (X, Y) right Bezout factors of (N, D) shows that $\tilde{D}_c F^{-1}$ is implied to be stable. As a result, $\tilde{D}_c F^{-1} = W$ with W any stable transfer function.

Now stability of F and FC implies stability of $W^{-1} \begin{bmatrix} \tilde{D}_c & \tilde{N}_c \end{bmatrix}$, which after postmultiplication with the left Bezout factors of $(\tilde{D}_c, \tilde{N}_c)$ implies that W^{-1} is stable. This proves that $F = W^{-1}\tilde{D}_c$ with W a stable and stably invertible transfer function.

$(b) \Rightarrow (a)$. Stability of F and FC is straightforward. Stability of $S_0 F^{-1}$ and $G_0 S_0 F^{-1}$ follows from (A.1), using the fact that $(\tilde{D}_c D + \tilde{N}_c N)^{-1}$ is stable (lemma A.1). □

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