Conditions of Handling Confounding Variables in Dynamic Networks

Arne Dankers, Paul Van den Hof,
Donatello Materassi and Harm Weerts

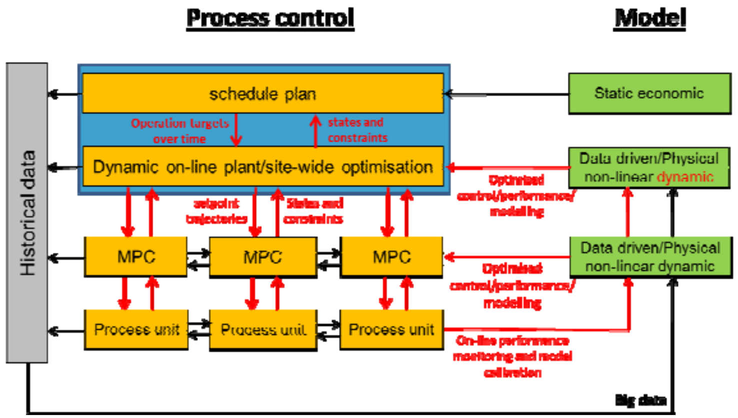
2017 IFAC World Congress, Toulouse, France, July 9-14

European Research Council
UNIVERSITY OF CALGARY
THE UNIVERSITY OF TENNESSEE KNOXVILLE
TU/e Technische Universiteit Eindhoven University of Technology

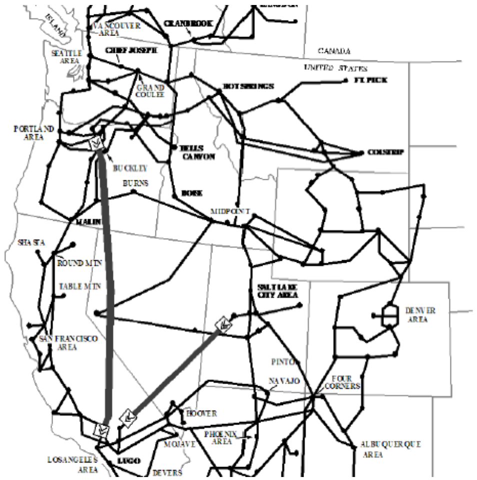
Where innovation starts

Introduction – dynamic networks

Decentralized process control

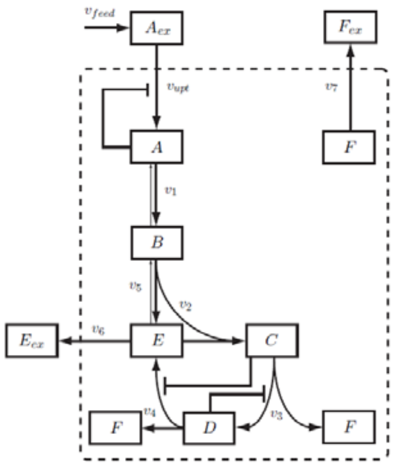


Power grid



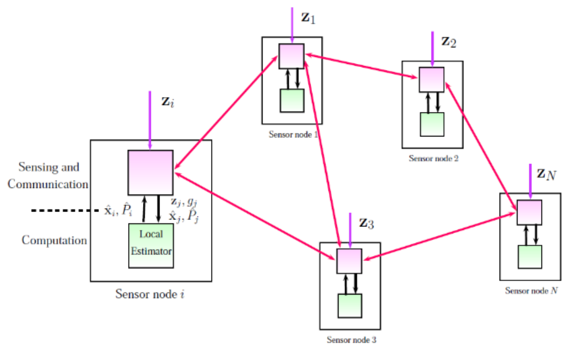
Pierre et al. (2012)

Metabolic network



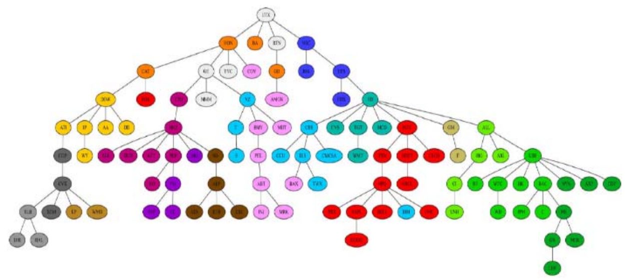
Hillen (2012)

Distributed control (robotic networks)



Simonetto (2012)

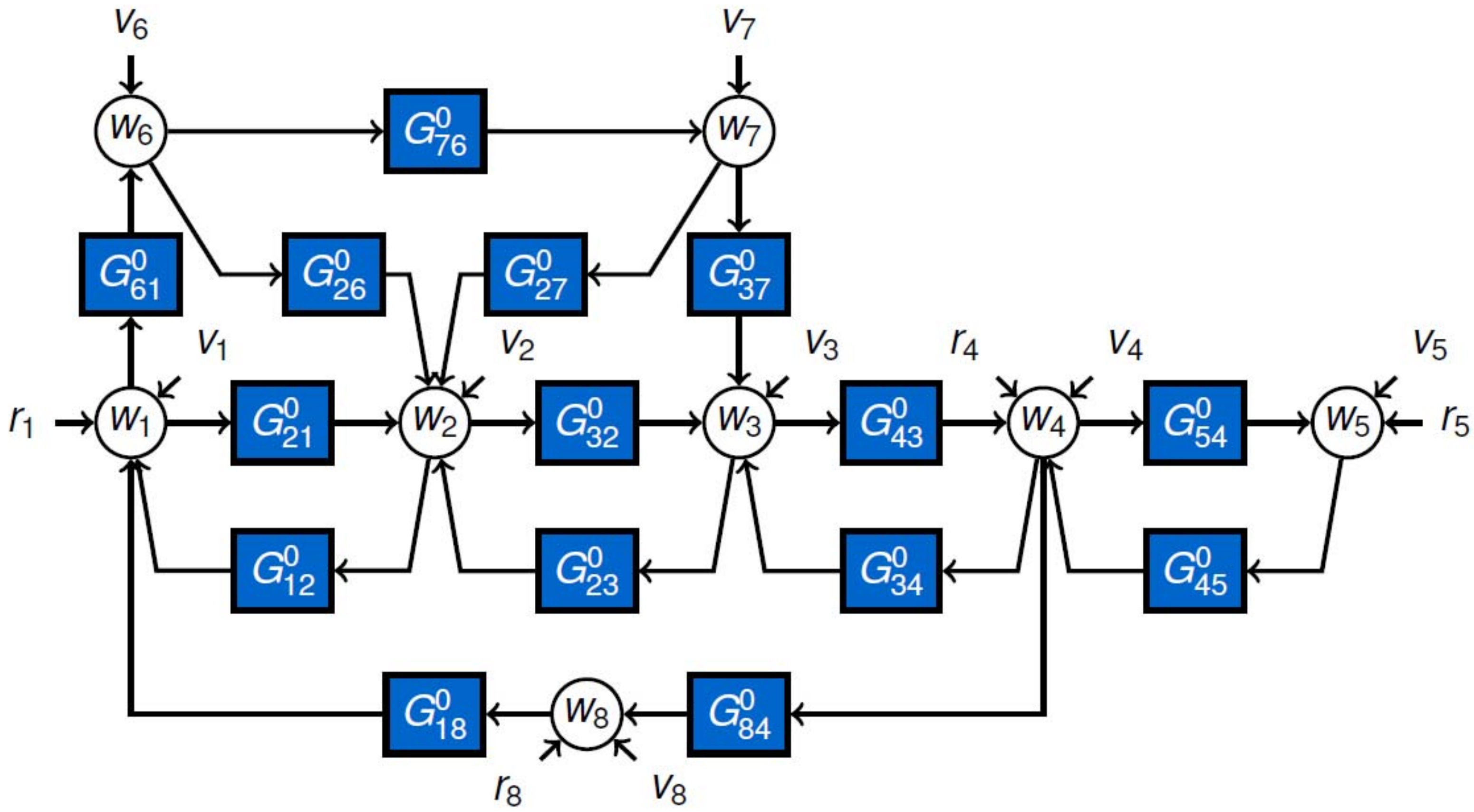
Stock market



Materassi et al. (2010)

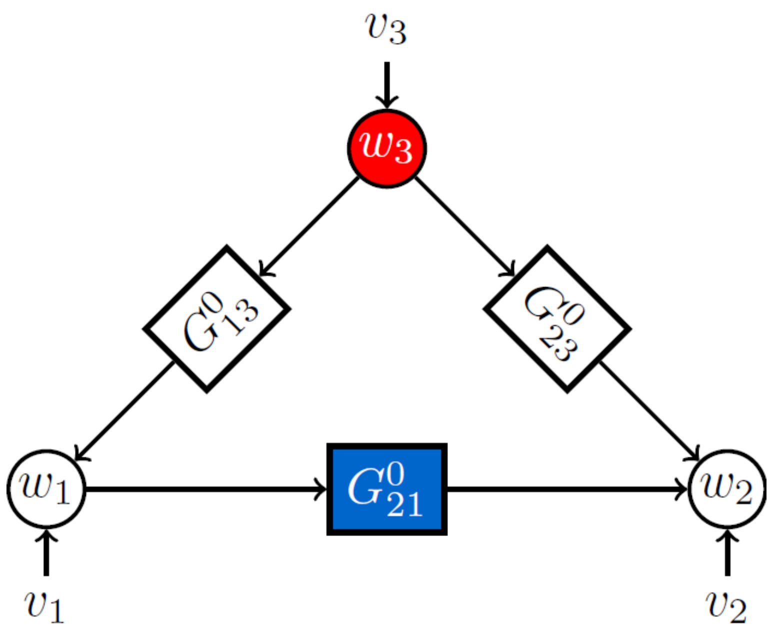
Technische Universiteit Eindhoven University of Technology

Identification in dynamic networks



Technische Universiteit Eindhoven University of Technology

Confounding Variables



- **Objective: Estimate G^0_{21}**
- **Suppose w_3 not measured**
- **w_2 is output**
- **w_1 is input**

Problem: Noise between input and output is correlated.

Estimate is not consistent.

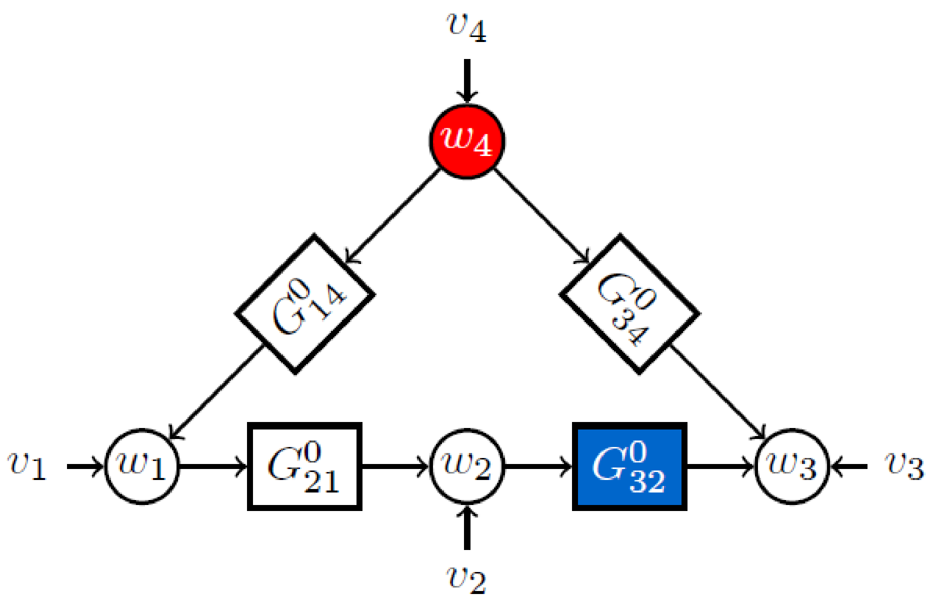
Then v_3 is a *confounding variable:*

- **Path from v_3 to output w_2, and**
- **Path from v_3 to input w_1**

that pass only through unmeasured nodes

Technische Universiteit Eindhoven University of Technology

Interesting Observation



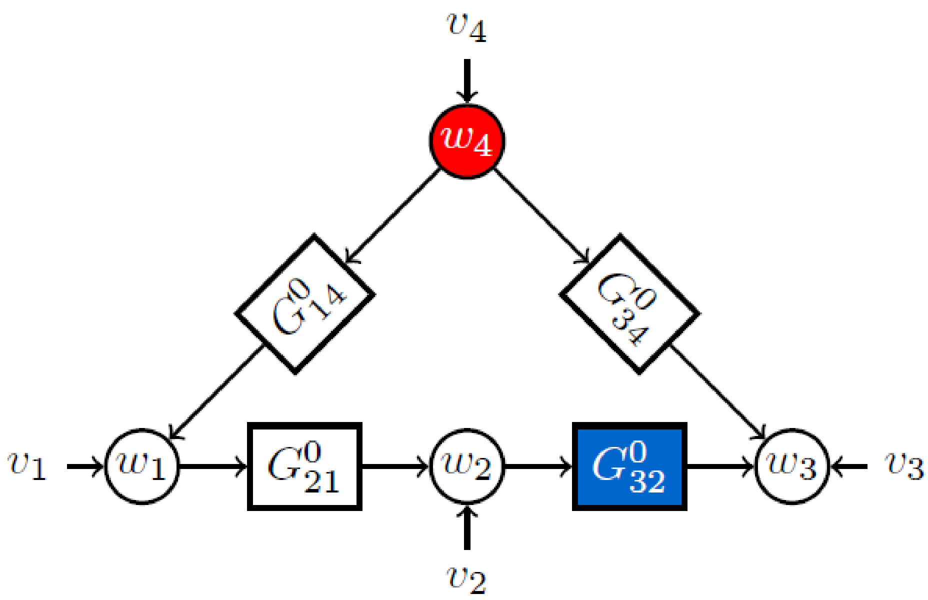
- **Objective: Estimate G^0_{32}**
- **w_2 is input**
- **w_3 is output**
- **Suppose w_4 is not measured (v_4 is a confounding variable)**

Observation:

Including w_1 as an additional predictor input results in consistent estimates of G^0_{32}.

TU/e Technische Universiteit Eindhoven University of Technology

Interesting Observation

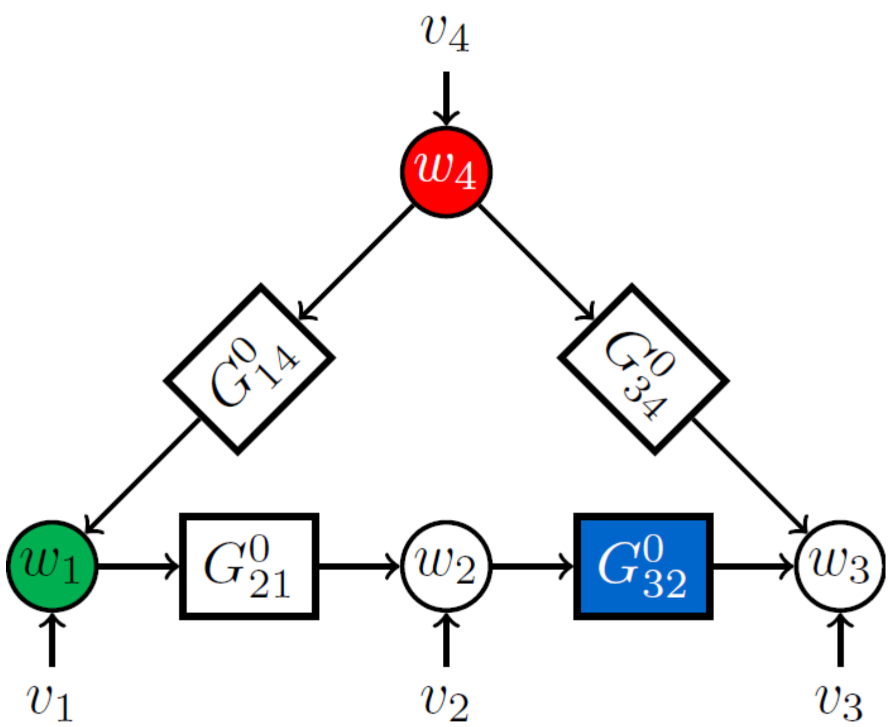


- **Objective: Estimate G^0_{32}**
- **w_2 is input**
- **w_3 is output**
- **Suppose w_4 is not measured (v_4 is a confounding variable)**

Questions:

- **Why does this work?**
- **Can we generalize this?**

Example Revisited



w_1 blocks the path from v_4 to w_2

→ w_2 can be partitioned as

$$w_2 = w_2^{(w_1)} + w_2^{(\perp w_1)}$$

dependent on v_4 | Independent of v_4

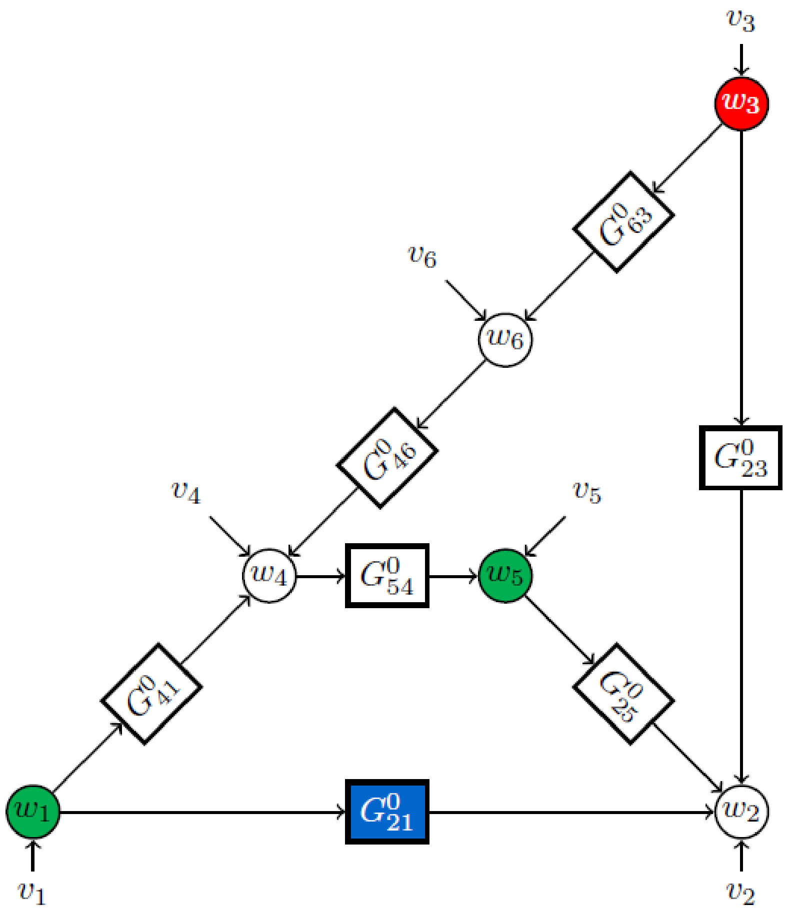
G^0_{32} can be consistently estimated using $w_2^{(\perp w_1)}$ as input, and w_3 as output (open loop identification problem)

Generalization

Consistent estimates of G_{ji}^{0} may be possible if all paths from confounding variables to predictor inputs w_k, $k \in A_j$ are blocked by additional inputs $w_n, n \in B_A$

Technische Universiteit Eindhoven University of Technology

Second Example



- **Objective: Estimate G^0_{21}**
- **First: selection of input nodes**
- **Parallel paths $w_1 \rightarrow w_2$ and loops around w_2 need to be blocked ***

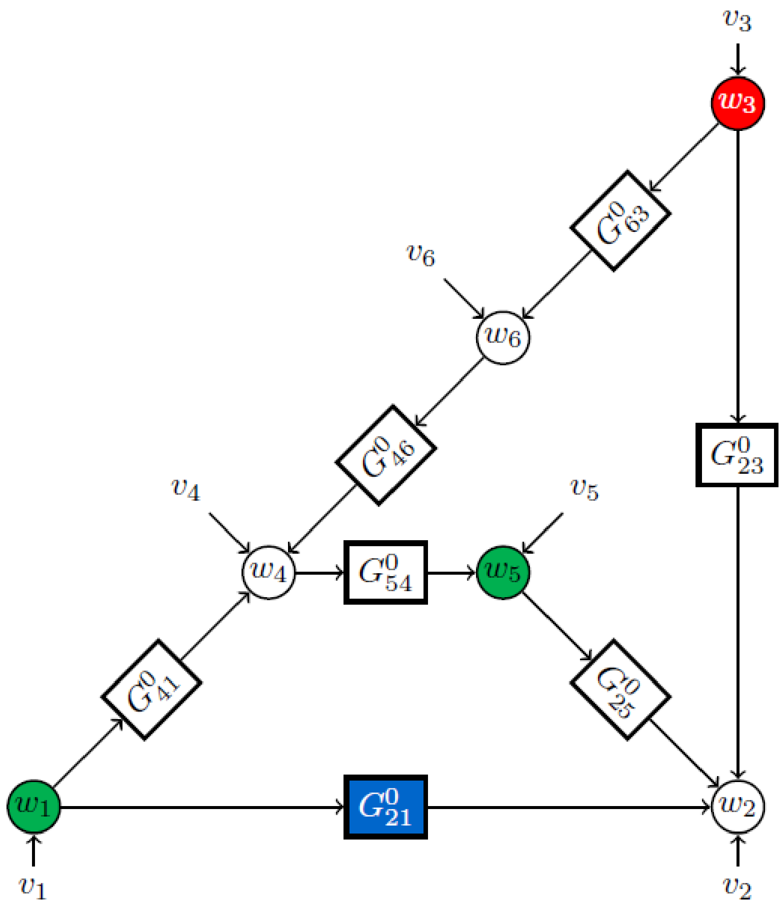
Select w_1 and w_5 as input

Suppose w_3 is not measured (v_3 is a confounding variable)

*** Dankers and Van den Hof (2014), Dankers et al., TAC, 2016.**

Technische Universiteit Eindhoven University of Technology

Second Example

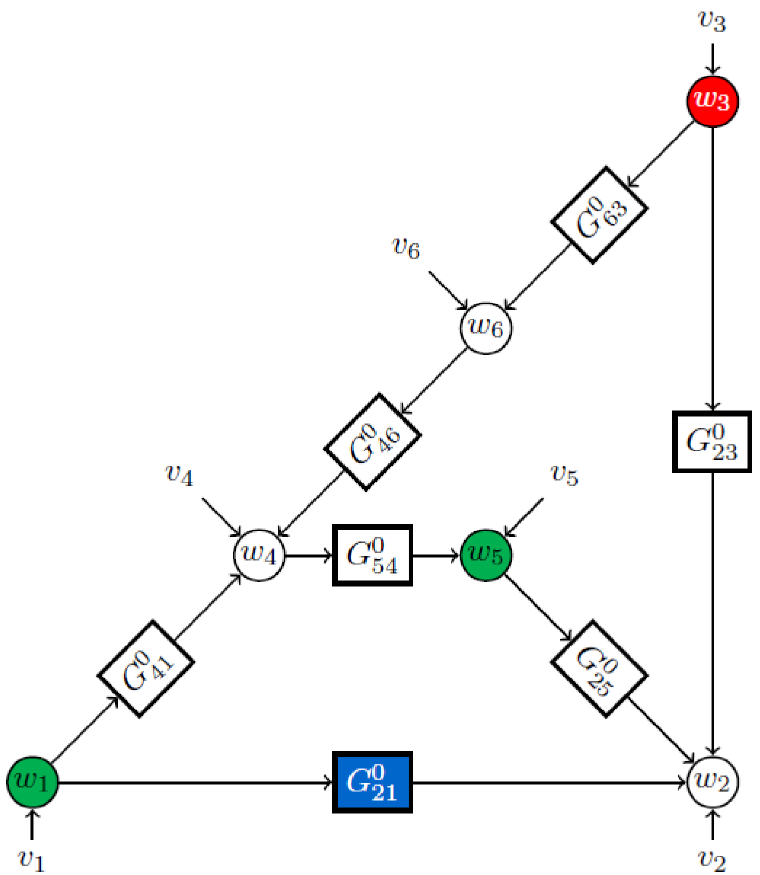


- **Objective: Estimate G^0_{21}**
- **To block path from $v_3 \rightarrow w_5$ can select either w_6 or w_4**
- **Try w_4, and partition w_5 as $w_5 = \underbrace{w_5^{(w_4)}}_{\text{dependent on } v_3} + \underbrace{w_5^{(\perp w_4)}}_{\text{Independent of } v_3}$**
- **Use $w_5^{(\perp w_4)}$ and w_1 as predictor inputs**

Does not work: $w_5^{(\perp w_4)}$ is independent of w_1.

$w_5^{(\perp w_4)}$ does not block parallel path from $w_1 \rightarrow w_2$

Second Example



- **Use w_6, and partition w_5 as $w_5 = w_5^{(w_6)} + w_5^{(\perp w_6)}$**

  $w_5^{(w_6)}$: dependent on v_3

  $w_5^{(\perp w_6)}$: Independent of v_3

- **In this case $w_5^{(\perp w_6)}$ is not independent of w_1**

  $w_5^{(\perp w_6)}$ blocks parallel path from $w_1 \rightarrow w_2$

Conclude: using $w_5^{(\perp w_6)}$ and w_1 as predictor inputs results in a consistent estimate of G^0_{21}!

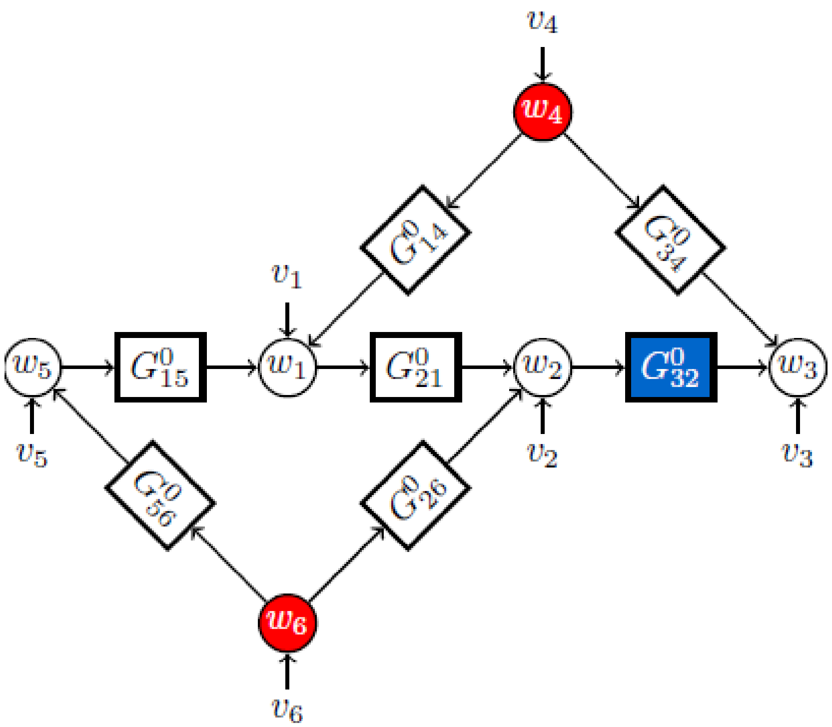
Second Condition To Handle Confounders

Generalization:

- **Let G_{ji}^{0} denote module of interest**
- **Let $w_k, k \in A_j$ denote the basic set of predictor inputs**
- **Then additional inputs $w_n, n \in B_A$ should not block:**
  - **any parallel paths from $w_i \rightarrow w_j$**
  - **any loops $w_j \rightarrow w_j$**

TU/e Technische Universiteit Eindhoven University of Technology

Third Example

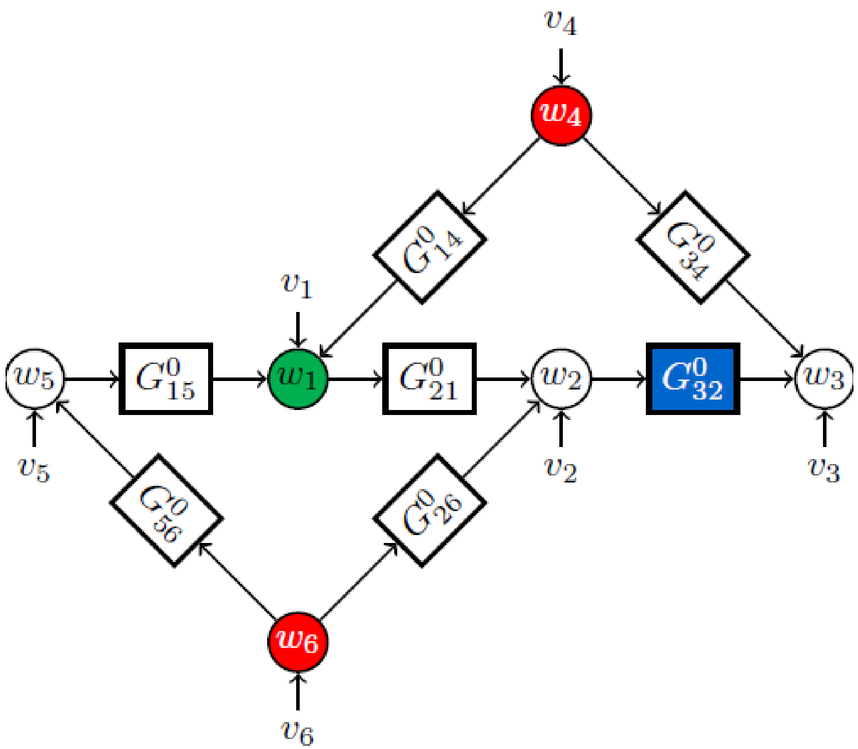


Objective: Estimate G^0_{32}

- **Choose w_2 as input**
- **w_3 is output**
- **Suppose w_4 and w_6 not measured (v_4 is a confounding variable)**

Technische Universiteit Eindhoven University of Technology

Third Example



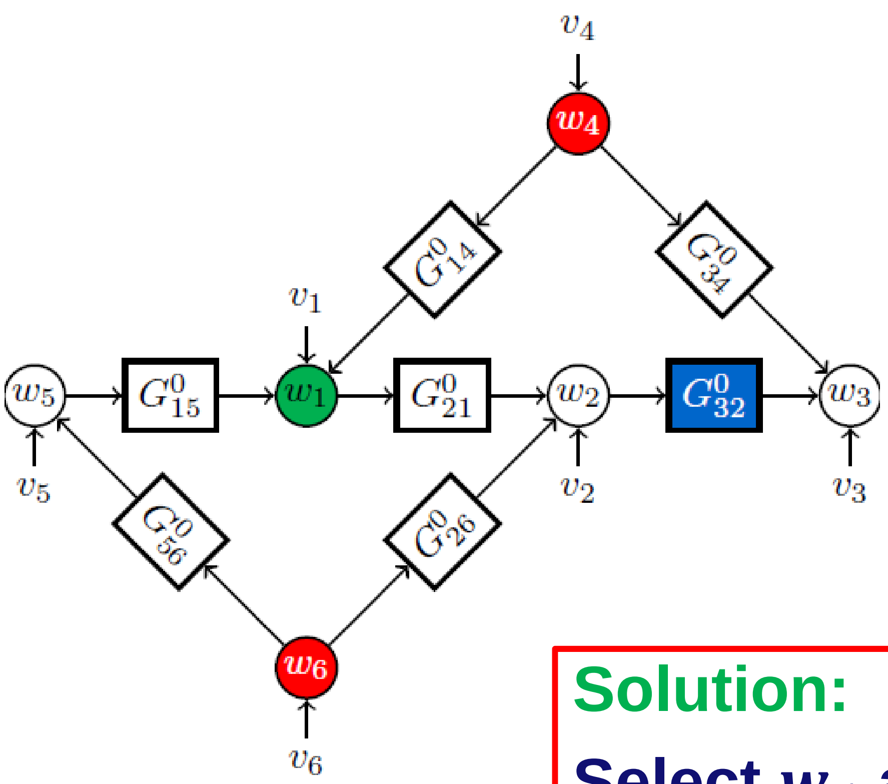
Objective: Estimate G^0_{32}

- **Choose w_2 as input**
- **w_3 is output**
- **Suppose w_4 and w_6 not measured (v_4 is a confounding variable)**

w_1 blocks path from $w_4 \rightarrow w_2$ ⇒ select w_1 as an additional predictor input.

TU/e Technische Universiteit Eindhoven University of Technology

Third Example



partition w_2:

$$w_2 = w_2^{(w_1)} + w_2^{(\perp w_1)}$$

Problem:

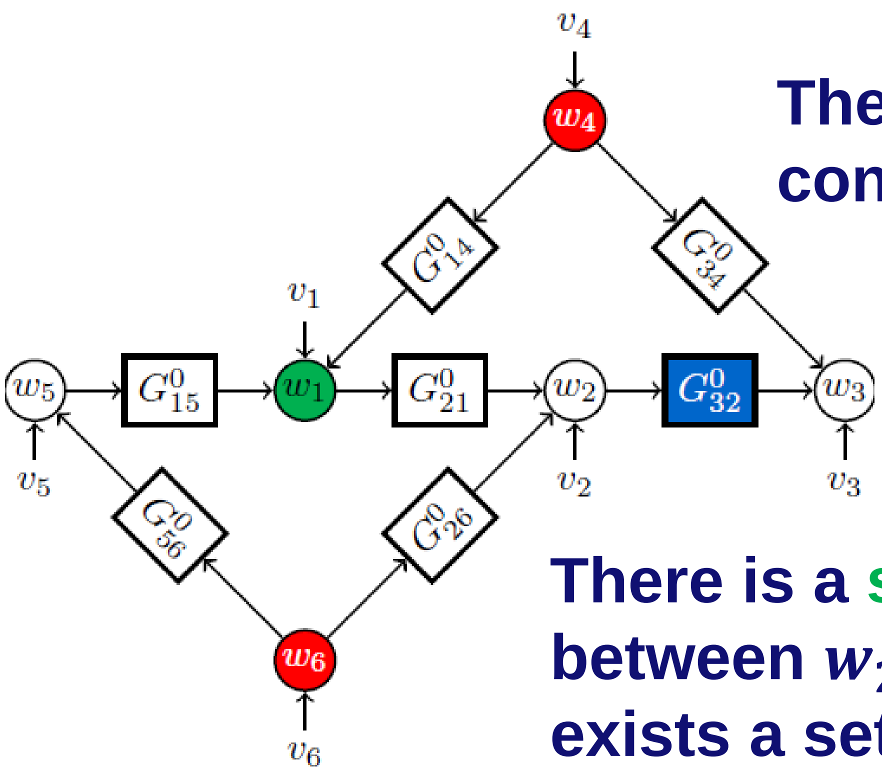
- **For partitioning w_2 we need to estimate G_{21}^0**
- **Estimating G_{21}^0 suffers from a "new" confounding variable v_6**

Solution:

Select w_5 as another additional input variable in order to consistently estimate the partitioning of w_2.

TU/e Technische Universiteit Eindhoven University of Technology

Sequence of Linked Confounders



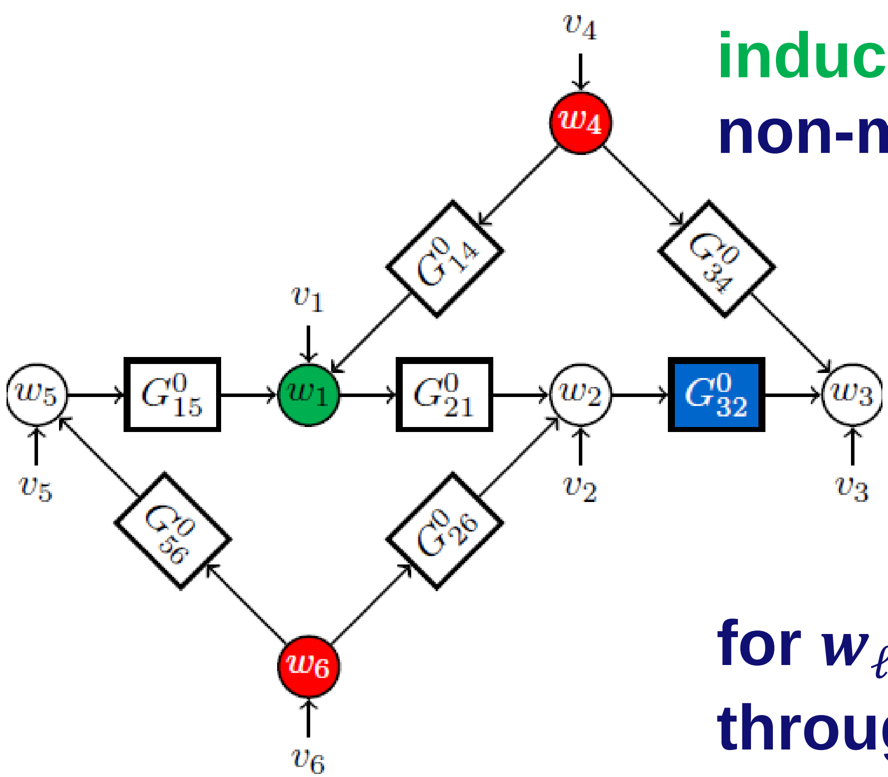
The variable w_1 links together two confounding variables.

There is a sequence of linked confounders between w_2 and w_3 induced by w_1 if there exists a set of non-measured nodes such that

$$v_{z_1} \to w_3, \quad v_{z_1} \to w_1$$
$$v_{z_2} \to w_1, \quad v_{z_2} \to w_2$$

only passing through non-measured nodes

Sequence of Linked Confounders



There is a sequence of linked confounders between w_i and w_j induced by B_A if there exists a set of non-measured nodes w_{z_i} such that

$$v_{z_1} \to w_j, \quad v_{z_1} \to w_{\ell_1}$$
$$v_{z_2} \to w_{\ell_1}, \quad v_{z_2} \to w_{\ell_2}$$
$$v_{z_3} \to w_{\ell_2}, \quad v_{z_2} \to w_{\ell_3}$$
$$\vdots$$
$$v_{z_n} \to w_{\ell_n}, \quad v_{z_n} \to w_i$$

for $w_{\ell_1} \in B_A$, and all paths passing through non-measured nodes only.

Implementation

Objective: estimate G_{ji}^0

1. Select variables that
   1. Block all parallel paths from $w_i \to w_j$
   2. Block all loops from $w_j \to w_j$

   Denote this set of variables $w_k, k \in A_j$
2. If there are confounding variables present, select a set B_A of additional variables that
   1. Block paths from confounding variables to w_i or w_j
   2. Does not block parallel paths or loops around w_j, and
   3. Does not induce a sequence of linked confounders
3. Minimize the prediction error:

$$\varepsilon(t,\theta) = H^{-1}(q,\theta)\left(w_j - \sum_{k\in D_j} G_{jk}(q,\theta) w_k \right),$$

where $D_j = A_j \cup B_A$

Conclusion

- **Confounding variables can be effectively handled by selecting additional measured predictor inputs**

- **Graph-property is closely related to the notion of d-separation for Directed Acyclic Graphs** (Pearl, 2000)

- **Alternative solutions** (Van den Hof et al., CDC 2017, submitted)