On the role of uncertainty models in identification for robust control

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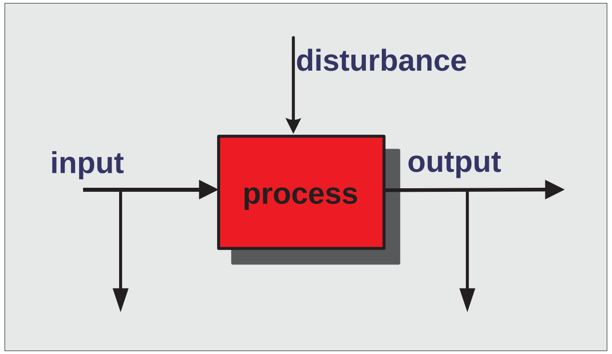
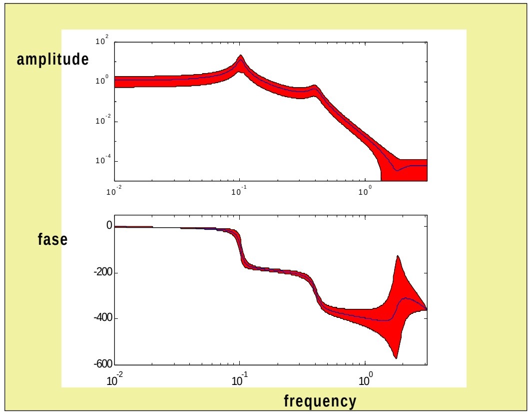


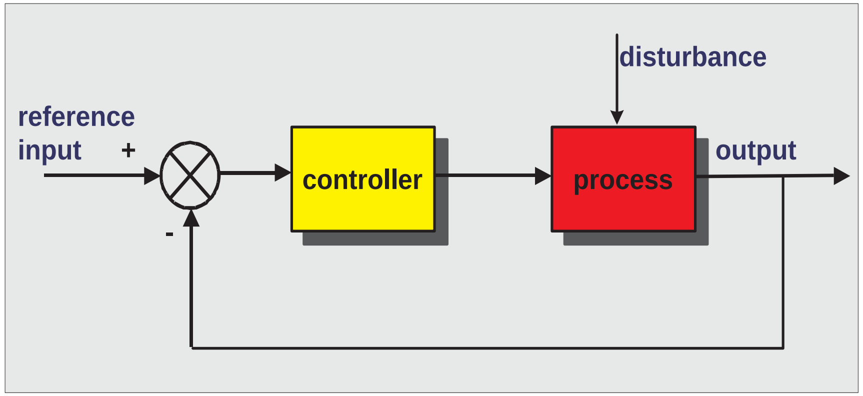
Identification

Data → Model



Feedback control system

Model → Controller



Relevance of model uncertainty

- Determines the achievable robust stability/performance $\rightarrow$ reduce uncertainty in control-relevant area
- Guidelines for appropriate identification after dedicated experiment design
  - experimental data and priors determine set of unfalsified models
  - identification technique determines nominal estimate
  - model uncertainty bound additionally determined by representation choice

Model uncertainty structures

In Control

- **unstructured additive, multiplicative ($\mathcal{H}_\infty$-norm bounded)**
- **real parametric**
- **Youla parameter**
- **gap, ν-gap metric**

In Identification

- **parametric uncertainty (statistical or worst-case)** *(e.g. Ljung (1987), Milanese et al. (1996), Bombois et al. (2001))*
- **additive frequency response bounds ($\mathcal{H}_\infty$-norm)** *(e.g. Goodwin et al. (1992), Hakvoort et al. (1997), Chen and Gu (2000))* **on open-loop or closed-loop plant**

Question to be considered:

Do robust stability/peformance requirements in a particular control problem motivate the use of a specific uncertainty structure in identification?

Is there a best uncertainty structure for identification?

In this presentation:

- **Some (relevant) thoughts and aspects**
- **Example for particular controller tuning problem**

Structures to be considered first

For G_x a nominal model:

- **gap, ν-gap**

$$\mathcal{G}_\delta(G_x, \delta_G) := \{G_\Delta \mid \delta(G_x, G_\Delta) \leq \delta_G\}$$

with δ either the gap or ν-gap metric

- **(dual) Youla parameter**

$$\mathcal{G}_Y(G_x, C, Q, Q_c, \gamma_G) := \{G_\Delta = (\bar{N}_x + \bar{D}_c\Delta)(\bar{D}_x - \bar{N}_c\Delta)^{-1} \mid \left\|Q_c^{-1}\Delta Q\right\|_\infty \leq \gamma_G\}$$

$C = \bar{N}_c\bar{D}_c^{-1}$ a present controller stabilizing $G_x = \bar{N}_x\bar{D}_x^{-1}$, Q, Q_c stable and stably invertible weighting functions.

Robust stability result -related to controller tuning

Given a nominal model G_x stabilized by C_x.
Consider two uncertainty sets around G_x chosen to incorporate the real plant G_0:

$\mathcal{G}_Y$ minimum sized dual-Youla model uncertainty set

$\mathcal{G}_G$ minimum sized gap-metric uncertainty set

Find corresponding (norm-bounded) sets of controllers around C_x that stabilize all models in $\mathcal{G}_Y$, resp. $\mathcal{G}_G$
e.g. controller retuning

Robust stability conditions - simultaneous perturbations

- **gap, ν-gap** *(Georgiou and Smith, Vinnicombe)*

$$\begin{aligned} \mathcal{G}_\delta(\delta_G) &= \{G \mid \delta(G, G_x) \leq \delta_G\} \\ \mathcal{C}_\delta(\delta_C) &= \{C \mid \delta(C, C_x) < \delta_C\} \end{aligned}$$

Robust stability if $\delta_G + \delta_C \leq \|T(G_x, C_x)\|_\infty^{-1}$

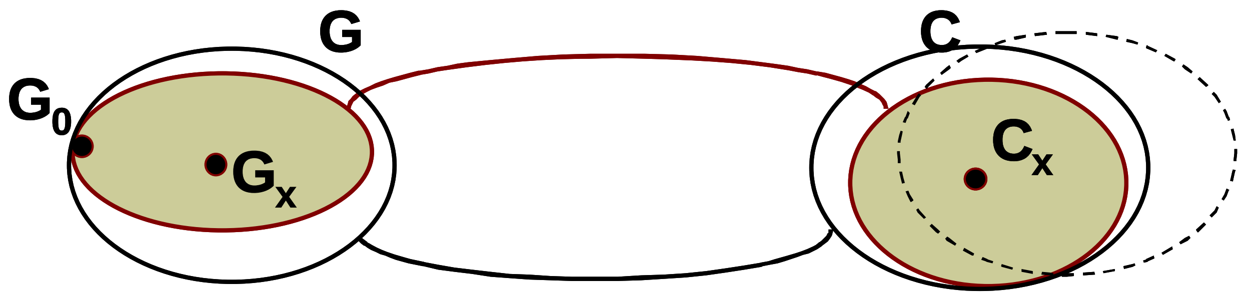
with $T(G, C) = \begin{pmatrix} G \\ I \end{pmatrix}(I + CG)^{-1}[C \quad I]$

- **Youla** *(Tay et al.)*

$$\begin{aligned} \mathcal{G}_Y(\gamma_G) &= \{(\bar{N}_x + \bar{D}_c \Delta_G)(\bar{D}_x - \bar{N}_c \Delta_G)^{-1}, \|\Delta_G\|_\infty \leq \gamma_G\} \\ \mathcal{C}_Y(\gamma_C) &= \{(\bar{N}_c + \bar{D}_x \Delta_C)(\bar{D}_c - \bar{N}_x \Delta_C)^{-1}, \|\Delta_C\|_\infty < \gamma_C\} \end{aligned}$$

Robust stability iff $\gamma_G \cdot \gamma_C \leq 1$

Result



$$\mathcal{C}_\delta \subset \mathcal{C}_Y \quad \text{for all } G_0, G_x$$

Result: The **Youla** set of controllers embeds the $(\nu)-$gap set of controllers, and therefore it is **less conservative**.

- The embedding property is not straightforward
- Crucial is the norm/metric-bounded property of the controller set

Theorem *Douma et al., 2001 - Automatica (2003)*

Given $\mathcal{G}_\delta(G_x, \delta_G)$, $\mathcal{C}_\delta(C_x, \delta_C)$ satisfying the (gap) robust stability condition, then

a) $\mathcal{G}_Y(G_x, C_x, I, I, \bar{\gamma}_G) \supseteq \mathcal{G}_\delta(G_x, \delta_G)$, for

$$\bar{\gamma}_G = \delta_G \|T(G_x, C_x)\|_\infty \left(1 - \delta_G \|T(G_x, C_x)\|_\infty\right)^{-1}$$

b) $\mathcal{C}_Y(G_x, C_x, I, I, \bar{\gamma}_C) \supseteq \mathcal{C}_\delta(C_x, \delta_C)$, with

$$\bar{\gamma}_C = \delta_C \|T(G_x, C_x)\|_\infty \left(1 - \delta_C \|T(G_x, C_x)\|_\infty\right)^{-1}$$

c) $\bar{\gamma}_G \cdot \bar{\gamma}_C \leq 1$, i.e. the two sets satisfy the Youla stability condition

Implication The Youla set of controllers embeds the $(\nu)-$gap set of controllers, and therefore it is less conservative.

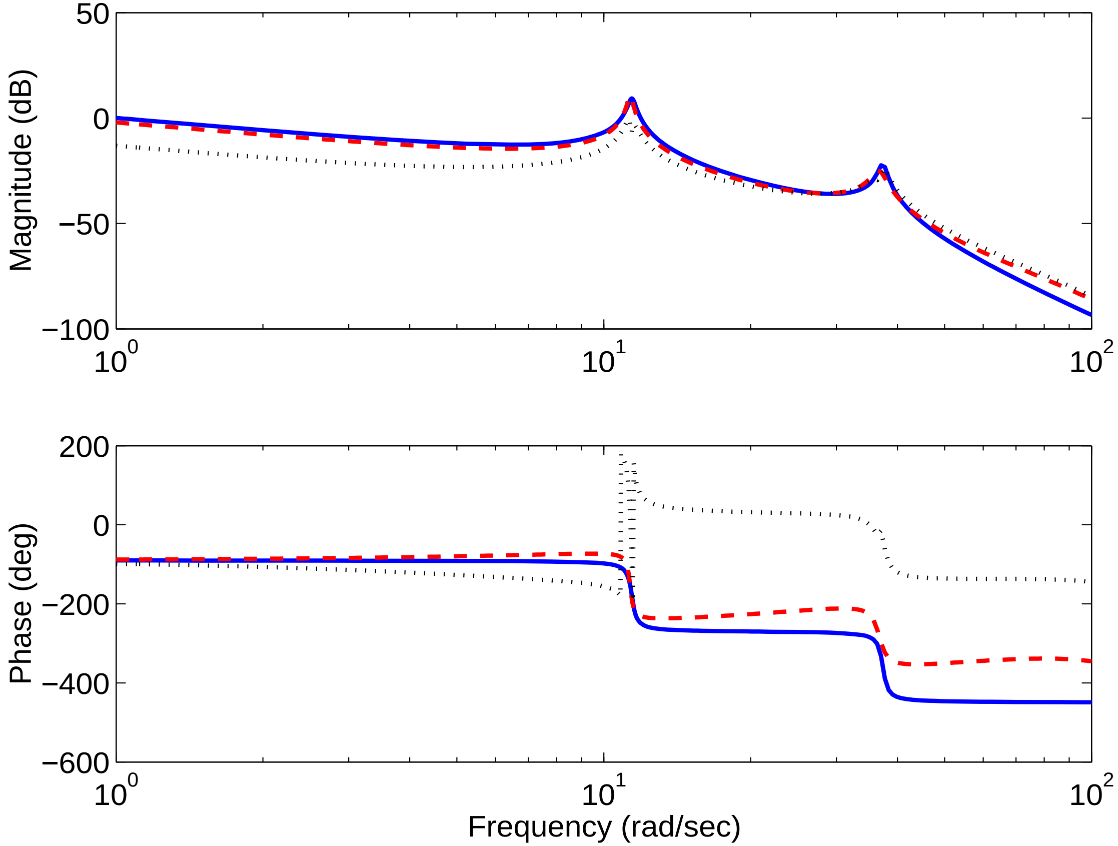
Example

Rotating disc system:

$$G_0 = \frac{182049.47}{s(s^2 + 0.377s + 130.5)(s^2 + 1.249s + 1395)}$$

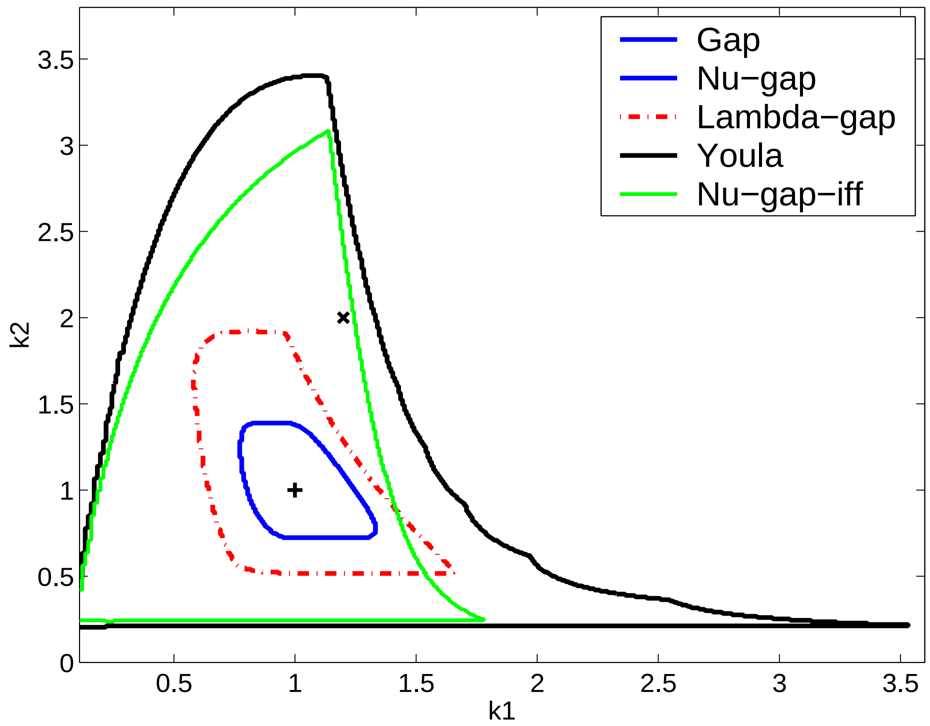
$$\hat{G} = \frac{0.0051602(s^2 + 70.53s + 1634)(s^2 - 45.72s + 1.608 \times 10^4)}{s(s^2 + 0.2671s + 128.3)(s^2 + 2.125s + 1348)}$$

$$C = \frac{1000}{(s+10)^3} \qquad C_{new} = \frac{1000k_2}{(s/k_1+10)^3}$$



$\boldsymbol{G_0}$, $\boldsymbol{G_x}$, **dotted:** $\boldsymbol{G_0 - G_x}$

Allowable range of controller perturbations according to robust stability conditions



$\|T(\hat{G}, C)\|_\infty^{-1} = 0.394$

$\delta(G_0, \hat{G}) = 0.235$

$\delta(C, C_{new}) = 0.417$

$\delta_\nu(G_0, \hat{G}) = 0.234$

$\delta_\nu(C, C_{new}) = 0.416$

$\|\Delta_G\|_\infty = 0.381$

$\|\Delta_C\|_\infty = 1.455$

$\|\Delta_G\| \, \|\Delta_C\| = 0.554$

$+$ **present controller** C;

$\times$ C_{new}, **designed to achieve a slighter larger bandwidth**

Contents

- **The question:**
  choice of uncertainty structure?
- **A particular case: controller tuning under robust stability**
- **Equivalence between structures - role of LFT's**
- **Conclusions**

Frequency domain sets (scalar)

- **Additive uncertainty set**

$$\mathcal{G}_a(G_x, W_a) := \{G_\Delta(s) \mid G_\Delta(s) = G_x(s) + \Delta_a(s) \ , \ |\Delta_a(i\omega)| \leq |W_a(i\omega)| \quad \forall \omega \in \mathbb{R}\}$$

- **Dual-Youla uncertainty set**

$$\mathcal{G}_Y(G_x, C, Q, Q_c, W_Y) := \left\{ G_\Delta(s) \mid G_\Delta(s) = \frac{\bar{N}_x(s) + \bar{D}_c(s)\Delta_G(s)}{\bar{D}_x(s) - \bar{N}_c(s)\Delta_G(s)} \ , \ \left| Q_c^{-1}(i\omega)\Delta_G(i\omega)Q(i\omega) \right| \leq |W_Y(i\omega)| \quad \forall \omega \in \mathbb{R} \right\}.$$

Both are (for each frequency) circular uncertainty regions in the complex plane

The same circular property appears to hold for

- ν-gap sets

$$\mathcal{G}_\nu(G_x, W_\nu) :=$$
$$\{G_\Delta(s) \mid \kappa\left(G_\Delta(i\omega), G_x\left(i\omega\right)\right) \leq |W_\nu(i\omega)| \quad \forall \omega \in \mathbb{R}\}$$

with κ the chordal distance

Linear fractional transformations map circles into circles:

$$F_u(P,\Delta) = P_{22} + P_{21}\Delta(1 + P_{11}\Delta)^{-1}P_{12} \ , \quad \text{with } \left|W^{-1}\Delta\right| \leq 1$$

can equivalently be described in an additive structure:

$$F_u\left(P,\Delta\right) = F_{centre} + \Delta_a \quad , \quad \left|W_a^{-1}\Delta_a\right| \leq 1,$$

with

$$F_{centre} = P_{22} + \frac{-P_{21}P_{12}P_{11}^{*}\left|W\right|^2}{1 - \left|P_{11}W\right|^2}$$

and

$$W_a = \frac{\left|P_{21}P_{12}\right|}{\left(1 - \left|P_{11}W\right|^2\right)}\left|W\right|$$

Result

Three model sets can equivalently be transformed into one another:

For Dual-Youla set:

$$\mathcal{G}_Y(G_x, C, Q, Q_c, W_Y) = \mathcal{G}_a(G_{centre}, W_a)$$

with

$$G_{centre} = C^{-1}\left(\frac{|N_c W_Y|^2}{|D_x|^2 - |N_c W_Y|^2}\right) + G_x\left(\frac{|D_x|^2}{|D_x|^2 - |N_c W_Y|^2}\right)$$

$$W_a = \frac{|\Lambda|}{|D_x|^2 - |N_c W_Y|^2}|W_Y|,$$

where $\Lambda = N_x N_c + D_c D_x$;
$N_x = \bar{N}_x Q$, $D_x = \bar{D}_x Q$, $N_c = \bar{N}_c Q_c$, $D_c = \bar{D}_c Q_c$.

For ν-gap set:

$$\mathcal{G}_\nu(G_x, W_\nu) = \mathcal{G}_a(G_{centre}, W_a)$$

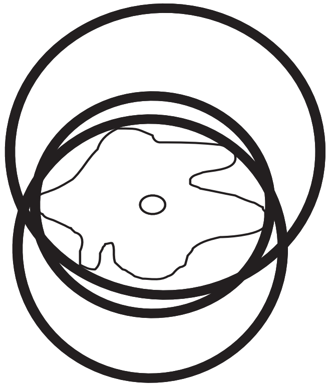
with

$$G_{centre} = \frac{G_x}{1 - \left(1 + |G_x|^2\right)|W_\nu|^2}$$

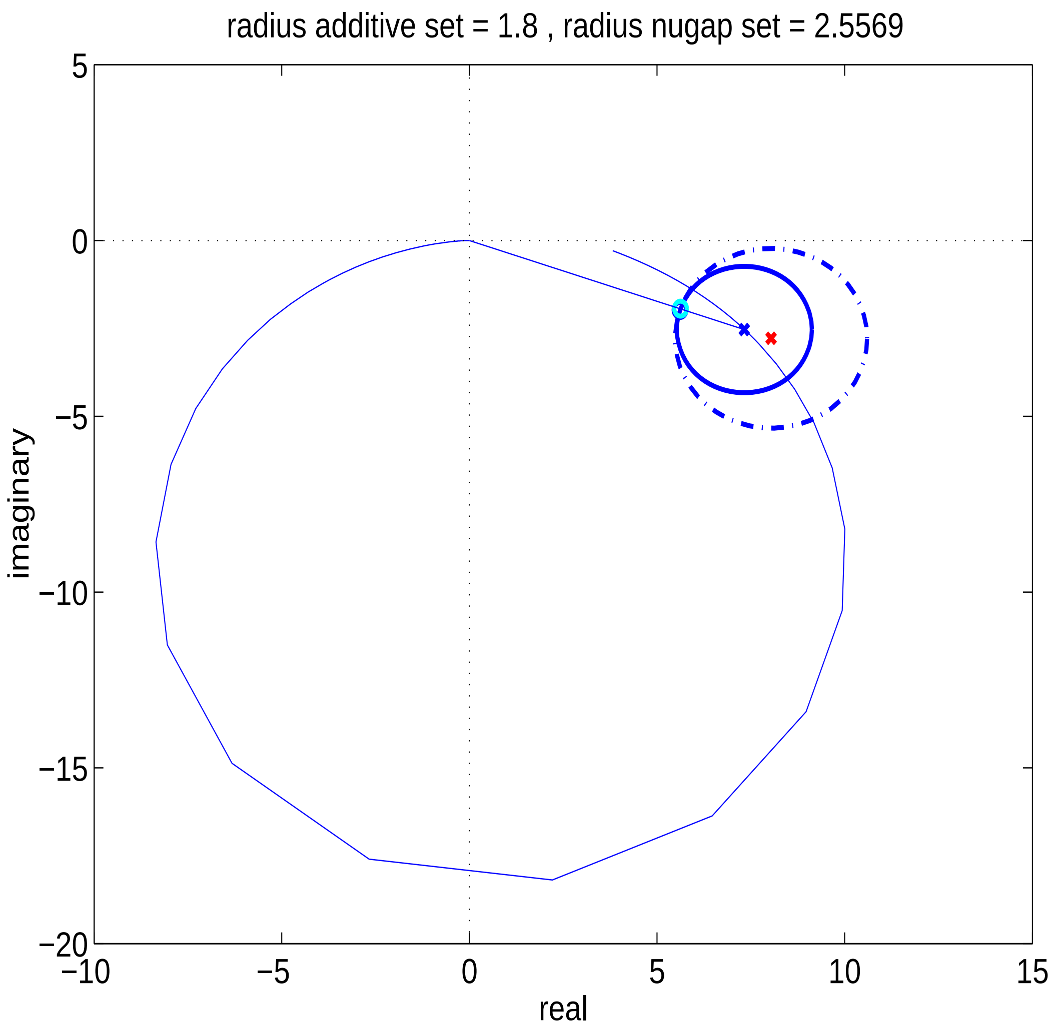
$$W_a = \frac{\sqrt{\left(1 - |W_\nu|^2\right)}\left(|G_x|^2 + 1\right)|W_\nu|}{1 - \left(1 + |G_x|^2\right)|W_\nu|^2}.$$

Observations from an identification perspective:

- **For identification of model uncertainty** *sets* **from data, the choice of structure "does not matter"**
- **Differences occur in complexities of G_{center} and weighting functions**
- **For a given/estimated nominal model $\hat{G} = G_{center}$ bounding the uncertainty in different structures leads to different results, affecting achievable robust performance**



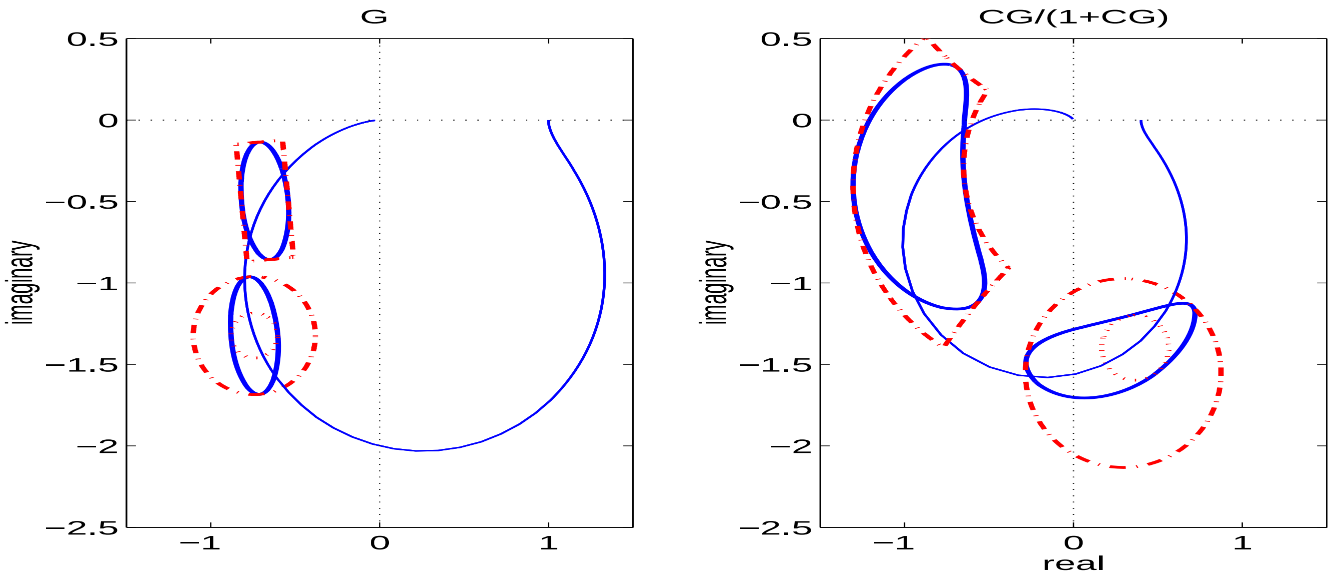
Embedding an additive nuncertainty set with a ν-gap set



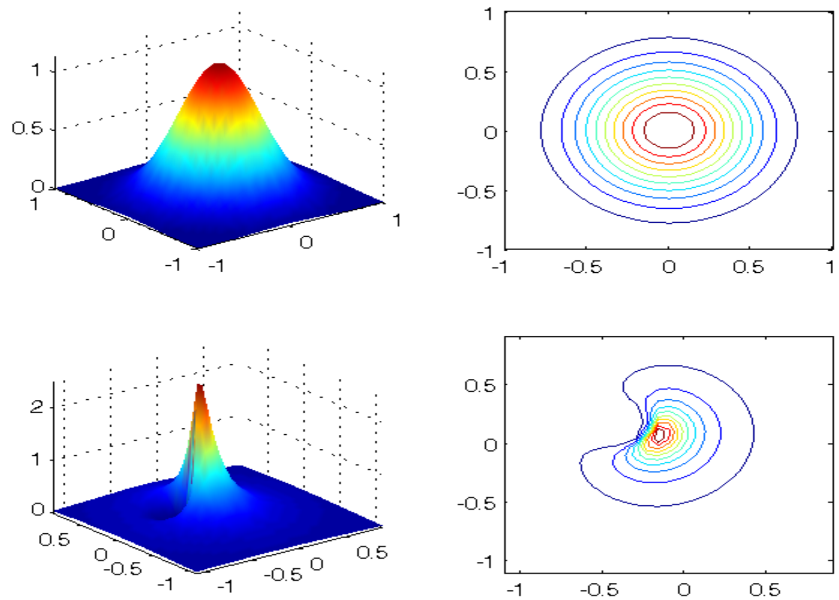
Non-circular bounds and probability density functions

For non-parametric uncertainty regions, e.g. confidence regions (ellipsoidal,boxed) in Nyquist curve, following a pdf:

General shapes are not maintained under LFT.



Non-circular bounds and probability density functions



Consequences (Heath 2000)

- probability density function changes
- unbiased estimate does not imply unbiased transform

Summary

- Attempt to address the question:
  How to choose an uncertainty structure in identification for robust control?
- Embedding results for two sets in a particular problem related to controller tuning
- LFT provides powerful tool to relate circular uncertainty sets
- If SYSID is split in (a) estimating nominal model and (b) bounding the uncertainty, the overall result can easily be non-optimal
- Transforms from open-loop to closed-loop model uncertainty sets (and vice versa): OK, but only for circular areas
- ... Work in progress